TEXT-BOOK

ON

HYDRAULICS

BY

GEORGE E. RUSSELL

ASSOCIATE PROFESSOR OF HYDRAULIC ENGINEERING
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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PREFACE

THE existence of so many works, of an excellent character, on the subject of Hydraulics demands that an explanation be offered for the appearance of a new one.

An experience in teaching has convinced the author that a book designed primarily for the classroom must generally be different in character from books intended for reference for practicing engineers. Into the latter class fall many of the admirable works now existent in Hydraulics, and the number of *bona fide* text-books is rather small.

The present volume is the result of the need at the Institute of a book suited for use in a number of courses where the amount of prescribed time, and ground to be covered, varies in each course. It seemed unnecessary that the book should treat of hydraulic motors, hydraulic machinery, and several other applications of hydraulics, since these subjects were being taught separately with the aid of books devoted to them exclusively. Therefore in this volume these topics have been omitted and the entire book devoted to a discussion of the more common and important problems.

An attempt has been made to so arrange the material that its order may be clear and logical, and its field of usefulness limited only by the ability of the instructor.

Much importance is placed upon the use of Bernoulli's Theorem as a fundamental in discussing problems of steady flow, since the writer has found that students are better able to grasp the problems of hydrodynamics once this principle has been made clear to them.

At the end of each chapter appear problems intended to illustrate the application of the principles just preceding, and in a majority of instances, references are given to standard literature in order that teacher, student, and practicing engineer may pursue their studies further if desired.

The author wishes to make special acknowledgment of his obligations to all those who have contributed to the making of this volume; to Professor I. P. Church of Cornell University for his kind permission to quote, from his "Hydraulic Motors," portions of the treatment there given of Water-hammer in Pipes; to Mr. J. T. Fanning of Minneapolis, Minn., for permission to use data on orifice and pipe flow appearing in his "Treatise on Hydraulic and Water Supply Engineering"; to Professor Dwight Porter of the Massachusetts Institute of Technology for helpful suggestions and the preparation of many of the numerical problems; finally, to Professor W. E. Mott of the Department of Civil Engineering, Carnegie Technical Schools, to whose encouragement, kind help, and friendly criticism the author owes much.

G. E. R.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, September, 1909.

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NOTE

In the present edition will be found, following the Appendix, a series of problems presented as a revision of those appearing at the ends of the chapters. These revised problems are temporarily placed there to avoid alteration in the plates which, under present war conditions, seems unwise.

August, 1918.

HYDRAULICS

CHAPTER I

INTRODUCTION

1. **Definitions.** — A fluid body is a mass of particles among which the force of cohesion is so slight that they may be easily made to change their relative positions by the application of small forces. We may conceive of a body as being entirely devoid of internal friction, thus permitting perfect freedom of motion of one particle past another. Between two particles the action of any force must, in this case, be in a direction *normal* to the contact surfaces, and the force cannot have any *tangential* component. Such a body we will call a "*Perfect Fluid*," although it is not known that one exists. Air, gases, water, alcohol, mercury, and many other bodies possess fluidity to a high degree and are often spoken of as perfect fluids. The extent to which the tangential component is present determines the ease with which the body may be made to change its shape. Oils, glycerine, molasses, etc., represent lesser degrees of fluidity.

Fluids may be either gaseous or liquid. The general study of the principles and laws which control the behavior of liquid bodies, when at rest or in motion, is generally referred to as "*Hydraulics*," although the derivation of the word signifies the flow of water in a pipe, and thus implies motion. There are naturally two divisions of the subject, *Hydrostatics*, a discussion of liquids at rest, and *Hydrodynamics*, a study of liquids in motion.

It is to be noted here that, while we shall confine ourselves in future discussions to the one liquid, water, the same laws and reasonings can be applied to any liquid whose physical

properties resemble those of water. For liquids at rest, the governing laws are independent of the degree of fluidity, since *motion* of the particles must take place before the tangential stresses can be developed. In the case of liquids in motion, this independence does not hold.

2. Physical Properties of Water. — Although water is not a perfect fluid, it is permissible for all engineering purposes to assume it so. Between its particles, therefore, the mutual pressures are always normal to the contact surfaces; and upon any plane or surface in the water the pressure must also be normal.

Weight. — The weight of water per unit volume varies with its temperature and purity. As one would expect, water near the boiling point is much lighter than at 39.3° Fahr., where it has its maximum density. Salt water, too, is known to be heavier than fresh, although the difference is not great. In Table I of the Appendix is given the weight of a cubic foot of distilled water at various degrees of temperature as determined by Rossetti. It will be noticed that for ordinary ranges of temperature the weight per cubic foot is not far from 62.4 lb.; and if we make some allowance for the presence of impurities, an average value of 62.5 lb. will suffice for ordinary calculations. In testing motors and pumps, where it is necessary to make accurate measurements of volume and weight, it is well to select from the above-mentioned table a unit weight to accord with the observed temperature of the water. For calculations involving salt water a mean value of 64.0 lb. may be assumed.

In subsequent formulæ in this book, the weight per cubic foot of water will be represented by w.

Compressibility. — Water has practically perfect elasticity of volume, but no elasticity of shape. It suffers sensible compression under great pressures, but regains its original volume upon removal of the pressure. Grassi found that a pressure of 14.7 lb. per square inch (one atmosphere) on each of the faces of a cube of water caused it to lose about 0.000050 of its original volume when its temperature was 32° Fahr. This is so small an amount as to almost justify the popular belief that water is incompressible, and for most engineering purposes

it may be so regarded. For pressures below 1000 lb. per square inch experiments indicate that the amount of compression varies directly as the pressure; but for very high pressures it has been recently pointed out by Hite* that this law no longer holds. Under a pressure of 65,000 lb. per square inch he obtained a reduction in volume of 10 per cent.

Example. — Assuming a cubic foot of sea water to weigh 64.0 lb. per cubic foot at the sea level, determine its unit weight at a depth of 500 ft.

Let w = unit weight at the surface.
w' = unit weight at 500 ft. depth.
V = volume of a given mass at surface.
V' = volume of same mass at 500 ft. depth.

The unit weight will vary inversely as the volumes, or

$$\frac{w'}{w}=\frac{V}{V'}.$$

By the law of Grassi

$$V' = (V - 0.00005\,V \times \text{ pressure in atmospheres}).$$

At a depth of 500 feet it will be shown later that the pressure is $\frac{500}{33.1}$ atmospheres. We may therefore write

$$w' = \frac{w \times V}{V - 0.00005\,V \times \frac{500}{33.1}},$$

or, since V in this particular example is 1 cubic foot,

$$w' = \frac{64}{1-\frac{0.025}{33.1}} = 64.05 \text{ lb. per cubic foot.}$$

From this it may be seen that the increase in weight due to compression is too slight to be regarded in ordinary computations.

Modulus of Elasticity. — On the basis of Grassi's figures we may determine a modulus of elasticity for water which will be

* Engr. News, Oct. 4, 1900.

a *volume* or *bulk* modulus. It is the value of the ratio of the stress per unit of area to the change per unit of volume. Denoting the former by p and the latter by $\Delta V \div V$, where ΔV represents the total change in volume, we have

$$E = \frac{p}{\Delta V \div V} = \frac{14.7}{.00005} = 294{,}000 \text{ lb. per square inch.}$$

It should be noted that this value of E holds only for pressures under 1000 lb. per square inch and for ordinary temperatures.

3. Equal Transmission of Pressure. — One of the marked characteristics of a fluid is its ability to transmit pressure, applied to its surface, equally in all directions and with undiminished intensity. Thus in the accompanying figure if a pressure of 10 lb. per square inch be applied to the water surface *ab* by means of a force exerted upon the piston, then this pressure will exist in *all parts* of the contained liquid and upon all parts of the reservoir walls in addition to any pressure that may previously have existed by virtue of the weight of the water itself.

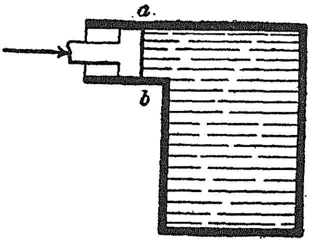

FIG. 1.

This principle finds an application in the hydraulic press. Here a comparatively small force, existing on the face of a piston in a small cylinder filled with water, is transmitted by a pipe to a larger cylinder, where it acts upon the face of a larger piston and exerts a total force in proportion to the face area. The ratio of this total force to that acting on the small piston will be as the ratio of the respective piston areas.

4. Units of Measure. — The elements of distance, force, and time enter into all hydraulic discussions, and will be measured by units of the foot-pound-second system. The foot is the common English foot of 12 inches, and the pound is the avoirdupois pound of 16 ounces. Unless otherwise stated, these will be the units employed in all hydraulic formulæ found in this work, and it is essential to see that all data are reduced to these

units before making substitution in the formulæ. In accordance with this plan, areas will be expressed in square feet; volumes, in cubic feet; intensity of pressure, in pounds per square foot; and duration of time, in seconds. It is quite common practice, however, to measure intensity of pressure in pounds per square *inch*, and volumes are often given in gallons. In this connection the following relations will be found useful:

1 cu. ft. = 7.48 U. S. gal.
1 U. S. gal. = 231 cu. in.
1 U. S. gal. = 0.8331 Imperial gal.

The Imperial gallon is mostly used in Great Britain.

As the motion of water, in most cases, is due to the action of gravity, the quantity g will often appear in this work. By g we represent the acceleration, due to gravity, of a freely falling body. It is the change in velocity during one second of time, and consequently will be expressed in feet per second, per second. Its value at any point on the earth's surface depends on the distance of the point from the center of the earth's mass, and may be obtained from the following formula by Pierce:

$$g = 32.0894\,(1 + 0.0052375 \sin^2 l)\,(1 - 0.0000000957\, e).$$

Here l is the latitude of the place and e its distance in feet above sea level. A mean value for g, and one sufficient for all ordinary calculations, may be stated as 32.2 ft. per second, per second.

5. Numerical Problems. — In order to familiarize one's self with the principles of hydraulics, no better way can be found than in the working of many numerical problems. To this end numerous problems will be given in this book, and should be worked out by the student. Habits of neatness, accuracy, and method should be acquired in their solution, and it is desirable that each problem be done in ink on a separate sheet of good paper, and finally bound into one convenient volume. Such a book will prove of great value to the student, not only in the training received while making it, but for reference in after years.

In general the use of logarithms is recommended, tables of four or five places being sufficiently accurate. The student is warned against deceiving himself as regards the accuracy of his results, and he should inspect his data with a view to determining the probable percentage of error in his answer. This latter should not be given so as to indicate a higher degree of accuracy than is warranted by the data. A problem may serve to illustrate the point. The cross-section of a natural stream is figured as 756.2 sq. ft., and the mean velocity of the stream past the section is 3.2 ft. per second as obtained by observations with a current meter. The discharge of the stream as given by the product of the above figures is 2419.84 cu. ft. per second. This result, however, is misleading, as the area may be in error by 10 sq. ft., and the figure for velocity in error by at least 0.2 ft. per second. If the area were 745 sq. ft. and the velocity 3.0 ft. per second, the rate of discharge would be 2235 cu. ft. per second, while with an area of 765 sq. ft. and a velocity of 3.4 ft. per second, the discharge would be 2601 cu. ft. per second. Between these two figures there is a wide difference, and our answer contains all the possible accuracy when stated as 2400 cu. ft. per second.

The use of a slide rule will often save laborious calculations, and the student should early acquire a knowledge of its use and limitations.

PROBLEM

1.—If a vertical cylindrical column of water is 100 ft. high, what would be its height (cross-section remaining constant) if water were absolutely incompressible?

CHAPTER II

HYDROSTATICS

6. Intensity of Pressure.—By intensity of pressure we denote the pressure per unit of area. If dA represents a very small part of an area A, and dP the total pressure upon it, then the intensity of pressure is

$$p = \frac{dP}{dA}.$$

If the intensity be the same for the entire area A, we may write the total pressure on A as

$$P = \int p \, . \, dA = pA,$$

from which

$$p = \frac{P}{A}. \quad \dots \dots \dots \quad (1)$$

The student will see that if p is not the same over the entire area, formula (1) gives an *average* value of p. As has been already pointed out, its numerical value in hydraulic work is commonly expressed in pounds per square inch.

7. General Relation between Pressures at Different Points in a Liquid.—In Fig. 2 let m and n be any two points arbitrarily chosen in the liquid whose upper or "free" surface is ab. Between m and n is shown any imaginary prism of water whose end faces contain the points in question. The cross-sectional area dA of the prism being infinitely small, the pressure on these faces may be assumed as of uniform intensity. Denoting these intensities by p_1 and p_2, the total end pressures are p_1dA and p_2dA respectively. The other forces acting upon the prism

are the pressures on its side faces and its own weight, which may be written $w \cdot l \cdot dA$. Since all these forces form a balanced system, they may be resolved into components along rectangular axes and the algebraic sum of each set put equal to zero. With axes as shown in Fig. 2, we may then write (remembering that the side pressures have no X comps.):

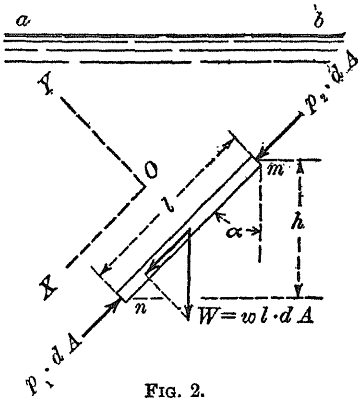

FIG. 2.

$$\Sigma X = p_1 dA - p_2 dA - w \cdot l \cdot dA \cos \alpha = 0$$

from which

$$p_1 - p_2 = wh. \quad . \quad . \quad (2)$$

h being equal to $l \cos \alpha$, the vertical distance between m and n. From (2) we see that the difference in pressure is dependent solely upon the difference in elevation between the two points.

Note. — If the points lie in the same horizontal plane, then the pressures are the same.

Had the point m been chosen in the surface ab, p_2 would have been the pressure of the atmosphere above the liquid, or p_a, and for the value of the pressure at any point lying a distance h below the surface, we should have

$$p = p_a + wh. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The appearance of p_a in this expression is another illustration of the transmission of pressure.

8. Illustration of the Foregoing Principles. — In Fig. 3 the reservoir M is connected by the tube CD with the reservoir at the lower level N. All portions of the tube are assumed filled with water.

The weight W on the piston at N is just heavy enough to maintain the water in M at a constant level. For the pressure at any point such as A we may write

$$p_1 = p_a + wh_1,$$

and for a point B at a depth h_2 below A the pressure is evidently p_1 increased by wh_2.

$$\therefore\ p_2 = p_1 + wh_2.$$

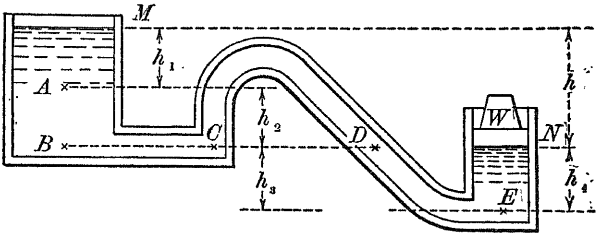

FIG. 3.

This value of p_2 must hold good for points C and D as they lie in the same horizontal plane. For a point E we have

$$p_3 = p_2 + wh_3,$$

and for the pressure at N, beneath the piston,

$$p_4 = p_3 - wh_4,$$

which may be written, by aid of the previous equation,

$$p_4 = p_a + wh_1 + wh_2 + wh_3 - wh_4,$$

or

$$= p_a + w(h_1 + h_2 + h_3 - h_4)$$

or

$$p_4 = p_a + wh.$$

This last might have been at once derived from equation (3), since h is the distance of the point at N below the free water surface.

9. Relative and Absolute Pressure. Pressure Head. — In the previous problem, p_4 is the pressure on each unit area of the under side of the piston at N, which is open to atmosphere on its upper side. It is clear that, as far as the effectiveness of the water pressure in sustaining the weight W is concerned, p_a need not enter into the discussion, as it acts on *both* sides of the piston and produces no resultant force. Eliminating p_a, we obtain a value for p_4 which measures the excess of the pressure over, or above, the atmospheric. We call this *Relative Pressure*.

If p_a be *included* in p_4, the pressure is measured above vacuum, or absolute zero, and is called *Absolute Pressure*. This latter need be seldom used in the simple problems of statics, as atmospheric pressure generally appears on a body as a set of balanced forces which produce only *internal* stress and may be discarded. Thus, if we are finding the pressure against a reservoir wall, we are not concerned with the absolute pressure, since the atmosphere is acting on the opposite face of the wall also, and has no effect upon its stability.

Equation (3), therefore, becomes, for most cases (using "relative" units),

$$p = wh. \quad \ldots \ldots \ldots \quad (4)$$

In either formula (3) or (4) it should be remembered that, in substituting numerical values, care must be taken to see that the units used are consistent. If w be in pounds per cubic feet, then h must be measured in feet and p will result in pounds per square foot. Similarly h may be in meters; w, the weight, in kilograms per cubic meter; and p will be in kilograms per square meter. The formula is therefore homogeneous, admitting the use of any system of units.

Pressure Head.—The quantity h has been used to represent the vertical distance from the free surface to the point in question. The distance is commonly called the *head* on the point, and because it causes the pressure p, it is also known as the *pressure head.* The same name is applied to its equivalent, $\frac{p}{w}$. If we are using *absolute* pressure units, the pressure head will be $(h + 34)$ ft., since atmospheric pressure is equivalent to that produced by a head of water measuring 34 ft. (See Art. 11.) Whenever the pressure (p_0) on the free surface varies from the atmospheric, the pressure head in absolute units will be the sum of the hydrostatic head and that corresponding to the pressure p_0. That is,

$$\frac{p}{w} = \left(h + \frac{p_0}{w}\right).$$

10. Definition and Form of a Free Surface.—We have already referred to the water surface in contact with the atmosphere as

a "free" surface. Properly speaking, if a liquid has a free surface, then upon that surface there is *no pressure.* It is common, however, to regard a surface upon which there is only atmospheric pressure as a free surface.

It is at once evident that a free surface is one of "equal pressure" and hence is horizontal (Art. 7).

11. Atmospheric Pressure and Water Barometer.—The atmosphere is a perfect fluid, exerting a normal pressure upon all surfaces with which it is in contact. It has been experimentally found that the intensity of its pressure, under normal barometer conditions at sea level, is about 14.7 lb. per square inch. Were it an incompressible fluid, we should find the pressure at a point distant h feet above sea level diminished by $w'h$, w' denoting the weight of atmosphere per cubic unit. Assuming this latter at 0.0807 lb., an experimental value found at freezing temperature under a pressure of 14.7 lb. per square inch, the pressure at an elevation of 6200 ft. above sea level would be

$$p = 14.7 - \frac{0.0807 \times 6200}{144} = 11.2 \text{ lb. per square inch.}$$

Actually the pressure is a little greater than this (11.6 lb. p.s.in.), owing to the variation in density of the atmosphere.

Water Barometer.—This very simple device may be used to demonstrate the existence of atmospheric pressure and also to measure its intensity. A long tube, closed at one end, is filled with water and the open end temporarily stopped. It is then placed in an inverted position with the stopped end beneath a water surface and the stopper removed. If the length of the tube be as great as 35 or 40 ft. it will be found that the water remains partly filling it, there being a portion of the tube above c (Fig. 4) which contains a partial vacuum. The absolute pressure at c is therefore zero and at d it is 14.7 lb. per square inch. From equation (2) we have

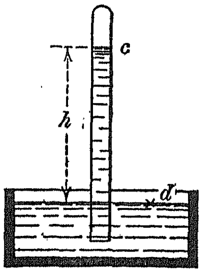

FIG. 4.

$$p_d - p_c = wh.$$
$$(14.7 \times 144) - 0 = 62.5\,h.$$
$$h = 34 \text{ ft. (approximately).}$$

This is the height to which a water column may be maintained by ordinary atmospheric pressure. If mercury be used in the tube instead of water, the height of the column will be

$$h = \frac{p_d - p_c}{w} = \frac{(14.7 \times 144) - 0}{848.7}$$
$$= 2.5 \text{ ft. or } 30 \text{ in.}$$

and the tube can be made shorter and more easy to handle. Church, in his "Mechanics of Engineering," points out the fact that the height of the water barometer is sensibly decreased by water vapor which forms, even at ordinary temperatures, in the top of the tube. This is not so in the case of mercury, save at high temperatures.

The operation of the so-called suction pumps depends on the above principle. A partial vacuum having been created by the piston or runner in the pump chamber, the water is forced by the atmosphere to rise in the suction tube; and, were the vacuum perfect, the limit to the height to which the water might be raised would be 34 ft. The impossibility of a perfect vacuum, however, coupled with the frictional resistance offered the moving water by the walls of the suction pipe, causes the distance to be materially reduced. The best of this type of pump can rarely exceed a lift of 28 ft. and a good "working-lift" is from 20 to 25 ft.

12. Water Piezometer. — A piezometer, as the name implies, is an instrument for the measuring of pressures. A water piezometer generally consists of a straight glass tube inserted in the side of a reservoir, containing water under pressure, and carried vertically upward to a height sufficient to prevent overflow (Fig. 5). The height h of the free surface in the tube above any point m in the reservoir exactly measures the pressure at that point, since $p = wh$. It is quite obvious that the location of the point of insertion makes no difference in the height h to which water will rise in the tube and ab marks the level of all such

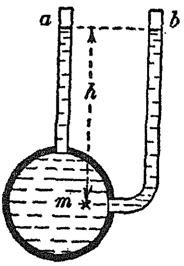

FIG. 5.

piezometer columns. As has been noted previously, the absolute head on the point m is $h + 34$ ft. since the top of the tube is open to the atmosphere. This form of piezometer is found very useful in experimental work, as the tube may be placed in any convenient position and connected with the reservoir by means of a piece of rubber tubing. Care should be taken in measuring small pressures that the internal diameter of the tube be large enough to prevent capillary action from affecting the height h. Gibson in his "Hydraulics" recommends the use of tubes having an internal diameter greater than 0.3 in., and figures that for this particular size the effect of capillarity is to increase the true height of the column by 0.15 in. (water at 68° Fahr.). For a 0.5-in. tube the increase is 0.092 in., and it would seem that no tube of smaller diameter should be used for accurate results.

If the pressure in the reservoir (Fig. 5) be maintained at less than the atmospheric, no column will rise in the piezometer and air will enter continuously at the top. If the top be bent over and downward into a vessel of water (Fig. 6), the atmosphere will cause a column of the water to rise to a height h in the tube, from which a measure of the pressure is obtained. Neglecting the weight of the air caught in the portion ab of the tube, the pressure on the free surface in the reservoir is the same as that at c. This latter, from (2), we know to be

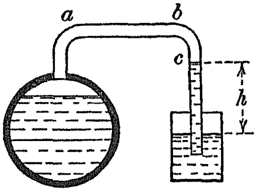

FIG. 6.

$$p = p_a - wh.$$

13. The Mercurial Gauge.—It will be seen that, for pressures differing much from atmospheric, the use of the above type of gauge necessitates a long length of piezometer tube. To obviate this, mercury is generally employed and the form of the tube changed. The general arrangement is shown in Fig. 7.

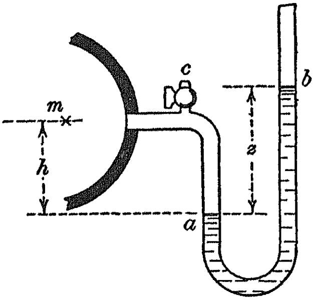

FIG. 7.

The tube is bent into a U-shape and into the loop of the U mercury is poured. A stopcock at c permits the expulsion of all air from the reservoir end of the tube, and the water may come into direct contact with the mercury. The difference in level z between the two mercury columns being indicative of the pressure at a, we have

$$p_m = w'z - wh,$$

or

$$p_m = w\,(13.59z - h),$$

13.59 being the specific gravity of mercury, or the ratio $\frac{w'}{w}$. If the position of the U-tube is such that a is *above* m, then the negative sign in the above expression becomes positive.

Mercury may be used also with much convenience in the vacuum gauge shown in Fig. 6. For the pressure on the free surface (either in the column or reservoir) we should obtain

$$p = p_a - 13.59\,wh.$$

14. The Differential Gauge. — This gauge is used to measure *differences* in pressure, and in general arrangement is similar to that shown in Fig. 8. A and B are two separate reservoirs connected by the tubing a-b-c-d-e-f. Mercury occupies the bottom of the U-shaped portion, and cocks at b and e permit the filling of the rest of the tube with water from the reservoirs. Because of greater pressure in A, there is a difference in level z between the two columns of mercury. From this measured difference we may proceed to calculate the difference in reservoir pressures as follows:

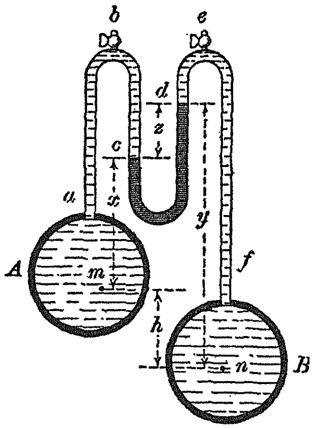

FIG. 8.

At c and d the pressures are $p_c = p_m - wx$ and $p_d = p_n - wy$ respectively, m and n being points arbitrarily chosen in A and B. The difference in these pressures is measured by the mercury column z so that

$$p_c - p_d = w'z = (p_m - wx) - (p_n - wy),$$

from which we may obtain

$$p_m - p_n = w'z + w(x - y).$$

Reference to the figure shows that

$$(x - y) = -(h + z),$$

and by substitution there results

$$\begin{aligned} p_m - p_n &= w'z - w(h + z) \\ &= w[z(s - 1) - h], \quad . \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

(s being the specific gravity of the mercury). If the points m and n be situated in the same horizontal plane, then $h = 0$ and (5) becomes

$$p_m - p_n = wz(s - 1) = 12.59\,wz. \quad . \quad . \quad . \quad (6)$$

This gauge finds a very wide use in practical and experimental work. Its one disadvantage, in the form as outlined, is that small differences of pressure are not possible of accurate measurement, since the mercury pressure-head is only one twelfth (roughly) of the actual differential head $\frac{p_m}{w} - \frac{p_n}{w}$. Any slight error, therefore, in reading the head z will be magnified in the same ratio. To obviate this it is common to use, in place of the mercury, a liquid whose specific gravity is only slightly greater than that of water. Cole in some of his experiments used a mixture of carbon tetrachloride and gasoline which had a specific gravity of 1.25. From (6) it will be seen that this resulted in the relation

$$\frac{p_m - p_n}{w} = .25\,z$$

$h = (S-1)z$
or $h = (1-S)z$

or z was *four times* as great as the actual difference in pressure heads. Therefore a small error made in measuring z was rendered negligible. The liquid most commonly used in place of the mercury is kerosene oil which has a specific gravity of approximately 0.79. Since it is *lighter* than water a slight modi-

:—
1. When $S < 1$, the gage magnifies.
2. When S is > 1, but < 2, "
3. " $S = 2$, it is neither.

fication of the gauge is necessary (Fig. 9). Cocks at a and b permit the partial filling of the vertical tubes ac and bd. Oil is then poured in at e until the upper portion of the tube is full and all air expelled. From the difference of level z we may obtain, by a process of reasoning similar to that already used,

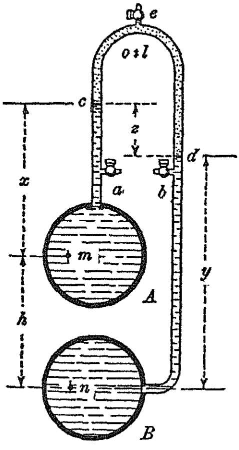

FIG. 9.

$$p_m - p_n = w[z(1-s) - h].$$

If $h = 0$, $\quad p_m - p_n = wz(1-s)$. $\quad$. (7)

Using 0.79 as the value of s for kerosene, there results

$$\frac{p_m - p_n}{w} = 0.21\, z.$$

15. Total Normal Pressure on Plane Surfaces. — If a plane surface be immersed in a horizontal position, it follows from Arts. 6 and 7 that the total normal pressure on it will be

$$P = Ap = Awh,$$

A being the area of the surface. It will now be shown that for a plane surface immersed in *any* position, the total normal pressure may be calculated from

$$P = Awh_0,$$

if h_0 be the head on the center of gravity of the surface.

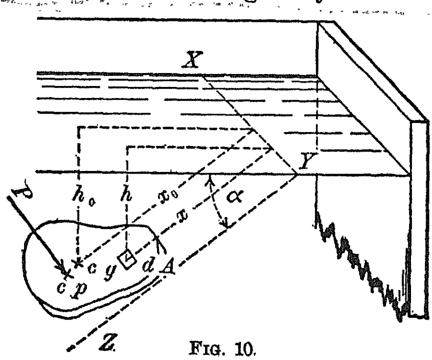

FIG. 10.

Figure 10 shows such a surface, lying in the plane XYZ

This plane cuts the water surface in the line XY, and the angle between the plane and the water surface we will call α. Selecting a very small part dA of the total area, so small that the pressure over it is of uniform intensity, we have, as the total pressure upon it,

$$dP = p \cdot dA = wh \cdot dA.$$

For total pressure on the entire area we have

$$P = \int wh \cdot dA = \int w \cdot x \sin \alpha \cdot dA = w \sin \alpha \int x \cdot dA.$$

The integral of $x \cdot dA$ is the moment of the area about XY as an axis, and may therefore be written $x_0 A$ where x_0 is the distance from the axis to the center of gravity of the area. We may therefore write

$$P = w \sin \alpha \; x_0 A,$$

or

$$P = Awh_0, \quad \dots \dots \dots \quad (8)$$

where h_0 is the head on the center of gravity. Hence the following theorem:

The total pressure on an immersed area is the product of that area, the weight of a cubic unit of water, and the head upon its center of gravity.

Example. — Find the total pressure on a vertical, rectangular sluice gate, 4 ft. wide by 6 ft. deep, the head on its upper edge being 10 ft.

$$P = Awh_0 = 4 \times 6 \times 62.5 \times 13 = 19{,}500 \text{ lb.} \quad \textit{Ans.}$$

16. Total Pressure on a Curved Surface. — It can be shown that the above theorem applies with equal exactness to any surface, be it plane, curved, or irregular. However, the total pressure on surfaces of the last two classes is usually of little or no practical value to the engineer. It would be the algebraic sum of a system of forces all acting in different directions, and in general we know that such a system cannot be replaced by a single resultant.

More often it is desired in the case of curved surfaces to find a *component* of normal pressure in some fixed direction. (See Art. 20.)

17. Center of Pressure. — An immersed *plane* surface is pressed upon by a system of parallel forces, infinite in number, which may be replaced by a single resultant force. The point on the surface at which this acts we will call the *Center of Pressure.* As noted in the previous paragraph, the pressures on a curved surface do not form a parallel system, hence they cannot, *in general*, be reduced to a single force.

The case of the plane surface may be represented by Fig. 10. The resultant of all the pressures on the surface is supposed to be acting at the point c. p. and we are to determine first its distance from XY, the line of intersection of the plane containing the surface and the plane of the water surface.

Considering a very small area dA, we have as the total pressure upon it,

$$dP = dA \cdot wh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

and its moment about XY is

$$dP \cdot x = dA \cdot wh \cdot x. \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

If in this way the moment of the pressure on each elementary area be found, we may place their sum equal to the moment of the resultant pressure by its arm x_c. That is,

$$P \cdot x_c = \int dP \cdot x,$$

or, from (a) and (b)

$$x_c \int dA \cdot wh = \int dA \cdot wh \cdot x. \quad . \quad . \quad . \quad . \quad . \quad (c)$$

From Fig. 10,

$$h = x \cdot \sin \alpha,$$

which substituted in (c) gives

$$w \cdot \sin \alpha \cdot x_c \int x \cdot dA = w \cdot \sin \alpha \int x^2 \cdot dA,$$

or

$$x_c = \frac{\int x^2 \cdot dA}{\int x \cdot dA} = \frac{I}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The integral of $x^2 \cdot dA$ will be recognized as the *Moment of Inertia* of the area, while the integral of $x \cdot dA$ is the expressed moment of the area about the chosen axis, or its *Statical Moment.*

$$X_c - X_o = \frac{I_g}{A v} = e$$

It is evident that x_c measures only the distance to the center of pressure down from XY and does not fix it in a lateral position. To do this it is necessary to take moments about another axis (not shown in Fig. 10) lying in the plane XYZ but at right angles to the previous axis. If by y we represent the distance from any elementary dA to this axis, we shall have as the moment of the total pressure on dA about this axis,

$$dP \cdot y = dA \cdot w \cdot h \cdot y;$$

and as before,

$$P \cdot y_c = \int dP \cdot y,$$

$$y_c \int dA \cdot w \cdot h = \int dA \cdot w \cdot h \cdot y,$$

$$h = x \cdot \sin \alpha,$$

and finally,

$$y_c = \frac{\int x \cdot y \cdot dA}{\int x \cdot dA}. \qquad \qquad (10)$$

In applying this formula, it is necessary that y be expressed as a function of x.

In general the engineer has only to do with the vertical position of the center of pressure and (10) is seldom used. For some special cases the lateral position is easily located, as will be shown in the following paragraphs.

Horizontal Surfaces.—If the surface be horizontal, the head on all points being the same, the forces are all equal, and the center of pressure will lie at the center of gravity.

18. Examples.—The forms of surfaces most commonly met with in engineering work are the rectangle, triangle, and circle. For these we will now derive general equations for locating the c. of p. Reference to formula (9) shows that the position of the c. of p. is independent of the angle α (Fig. 10) provided it has any value other than zero. (For this value the given derivation of (9) would not hold.) Hence we may rotate the surface about XY without the c. of p. changing. If $\alpha = 90°$, the h for each elementary area becomes its x also, and it will be a matter of convenience if we assume our surfaces vertical.

Rectangle (Fig. 11).—Here we will take our elementary area as a horizontal strip. Then all parts of it will have the same x.

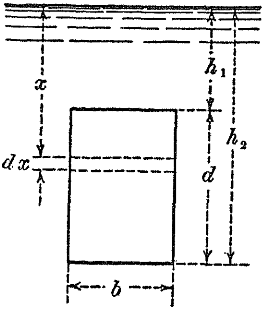

FIG. 11.

$$x_c = \frac{\int x^2 \cdot dA}{\int x \cdot dA}, \text{ where } dA = b \cdot dx.$$

$$\therefore\ x_c = \frac{\int_{h_1}^{h_2} bx^2 \cdot dx}{\int_{h_1}^{h_2} bx \cdot dx} = \frac{2}{3} \cdot \frac{h_2^3 - h_1^3}{h_2^2 - h_1^2}.$$

If $h_1 = 0$, or the upper edge of the rectangle be in the water surface,

$$x_c = \frac{2}{3} d.$$

This is a very useful relation and should be remembered.

Triangle. Base Horizontal, Vertex Up (Fig. 12). —

$$dA = u \cdot dx.$$

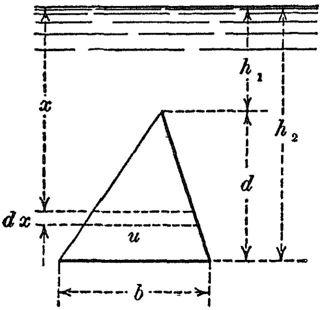

FIG. 12.

From similar triangles

$$\frac{u}{b} = \frac{x - h_1}{h_2 - h_1} \text{ or } u = \frac{b(x - h_1)}{h_2 - h_1},$$

$$x_c = \frac{\int x^2 \cdot dA}{\int x \cdot dA} = \frac{\int_{h_1}^{h_2} x^2(x - h_1)\frac{b}{h_2 - h_1}\, dx}{\int_{h_1}^{h_2} x(x - h_1)\frac{b}{h_2 - h_1}\, dx},$$

$$x_c = \frac{\frac{b}{h_2 - h_1}\left|\frac{x^4}{4} - \frac{h_1 x^3}{3}\right|_{h_1}^{h_2}}{\frac{b}{h_2 - h_1}\left|\frac{x^3}{3} - \frac{h_1 x^2}{2}\right|_{h_1}^{h_2}};$$

or

$$x_c = \frac{1}{2}\,\frac{(h_2 - h_1)^2(3\,h_2^2 + 2\,h_1 h_2 + h_1^2)}{(h_2 - h_1)^2(2\,h_2 + h_1)}$$

$$x_c = \frac{1}{2} \cdot \frac{3\,h_2^2 + 2\,h_1 h_2 + h_1^2}{2\,h_2 + h_1}.$$

If $h_1 = 0$, or the vertex lie in the water surface, then

$$x_c = \tfrac{3}{4}\, h.$$

Triangle. Base Horizontal, Vertex Down (Fig. 13). — By the same method as before we may arrive at the result

$$x_c = \tfrac{1}{2} \cdot \frac{3\,h_1^2 + 2\,h_2 h_1 + h_2^2}{2\,h_1 + h_2}.$$

If $h_1 = 0$,

$$x_c = \tfrac{1}{2} \cdot d.$$

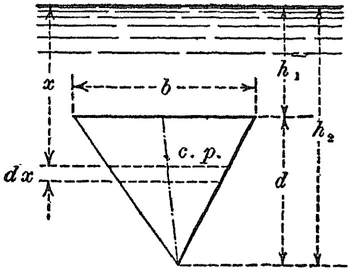

FIG. 13.

Note. In the case of the rectangle and triangles our elementary areas had their length parallel to the surface. This means that the pressure on each strip was uniformly distributed and might be replaced by a single force acting at the middle of the strip. Conceiving the whole area to be subdivided into elements, each having its resultant force, we find these latter lying on a *medial* line, as does also their resultant. In these cases the c. of p. is exactly located by formula (9) and the medial line.

Corollary. — If an area have an axis of symmetry at right angles to the moment axis, the c. of p. will lie upon it.

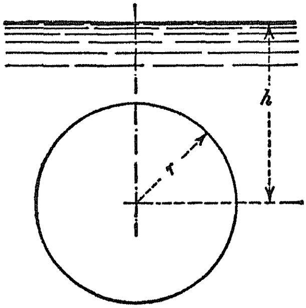

FIG. 14.

Circle. (Fig. 14). — Here let h be the head on the center of the circle.

$$x_c = \frac{\int x^2 \cdot dA}{\int x \cdot dA} = \frac{I_0 + Ax_0^2}{Ax_0},$$

or

$$x_c = \frac{\dfrac{\pi\, r^4}{4} + \pi\, r^2 h^2}{\pi\, r^2 h} = h + \frac{r^2}{4\,h}.$$

Example. — A trapezoid of the shape shown in Fig. 15 is immersed vertically with a head of 10 ft. on its upper edge. Find the location, both vertically and laterally, of the center of pressure.

For convenience divide the figure into a rectangle and triangle as shown, whose values of I and S may be found separately.

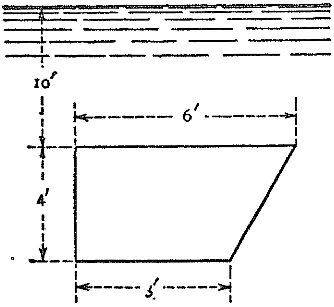

FIG. 15.

For the rectangle,

$$I = I_0 + Ax_0^2,$$

where I_0 is the moment of inertia of the figure about an axis through its center of gravity parallel to the water surface. The quantity x_0 is the distance between these two axes.

$$\therefore I = \frac{3 \times (4)^3}{12} + (12 \times 144) = 1744.$$

$$S = 12 \times 12 = 144.$$

Similarly for the triangle,

$$I = \frac{3 \times (4)^3}{36} + [6 \times (11\tfrac{1}{3})^2] = 771.$$

$$S = 6 \times 11\tfrac{1}{3} = 68.$$

Finally, $$x_c = \frac{I}{S} = \frac{1744 + 771}{68 + 144} = 11.9 \text{ ft.}$$

The lateral position of the c. of p. may be determined from the fact that it must lie on the medial line. (See note in preceding paragraph.) If y be its distance from the left hand edge of the figure, then

$$\frac{y}{3} = \frac{6.1}{8},$$

or $$y = 2.3 \text{ ft.} \quad \textit{Ans.}$$

19. Relation between Center of Gravity and Center of Pressure. — The position of the c. of p. is always *below* the c. of g. of a surface. Were the intensity of pressure *uniform* over the surface, the resultant pressure would necessarily be applied at the c. of g. But since the pressures increase in magnitude as the head increases, the resultant pressure is carried downward and is always applied *below* the c. of g. If we pass from moderate to

great depths, the variation in pressure on a surface becomes less and the c. of p. approaches the c. of g. as a limit. On small areas, therefore, located at depths of considerable magnitude, it is quite permissible to consider the positions of these two points as identical.

Example. — Find the position of the c. of p. for the rectangle shown in Fig. 16.

By Art. 17, $x_c = \frac{I}{S}$,

$$I = I_0 + Ax_0^2 = \frac{2 \times 4 \times 4 \times 4}{12} + (8 \times 10000)$$

$$= 10.667 + 80000.$$

$$S = 8 \times 100 = 800,$$

$$x_c = \frac{10.667 + 80000}{800} = 100 + .013.$$

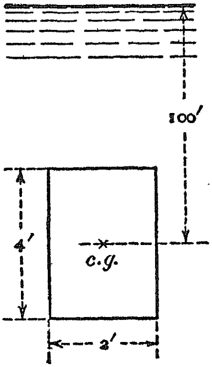

FIG. 16.

20. Component of Normal Pressure in a Given Direction. — Instead of finding the *total normal pressure* on a surface, it may be desired to find a component of this pressure parallel to some fixed direction. There arise several distinct cases.

Case (1). *Plane Surface.* — In Fig. 17, P_1 may represent the required component, making an angle α with the normal pressure P. The other component of P is, of course, perpendicular to the required component. For the value of P_1 we have

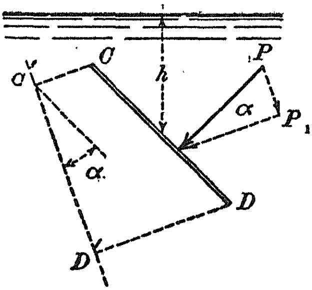

FIG. 17.

$$P_1 = P \cdot \cos \alpha = A\, wh_0 \cos \alpha. \quad (a)$$

The area A is represented by C-D in edge view and if C'-D' be the projection of A on a plane perpendicular to P_1, we have as a value of this projected area,

$$A_1 = A \cos \alpha,$$

from which and (a),

$$P_1 = A_1 wh_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

or P_1 is the product of the projected area and the intensity of pressure at the c. of g. of the original area. Hence the following

Theorem. — *To find the component of pressure parallel to a given direction, project the area upon a plane perpendicular to the component and multiply the projection by the intensity of pressure on the c. of g. of the original area.*

Example. — Let it be desired to find the horizontal and vertical components of total normal pressure against the surface *a-b*, shown in Fig. 18, assuming that the length of *a-b* measured perpendicular to the plane of the paper is 10 ft.

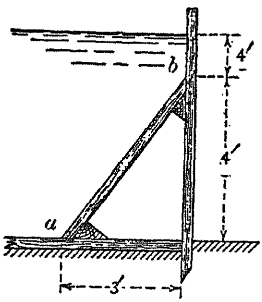

FIG. 18.

By the above theorem,—

$$P_v = (3 \times 10) \times 62.5 \times 6 = 11250 \text{ lb.}$$

$$P_h = (4 \times 10) \times 62.5 \times 6 = 15000 \text{ lb.}$$

Case (2). *Curved and Irregular Surfaces.* (Fig. 19). — The following figure and demonstration are given for the special case of a surface of single curvature,

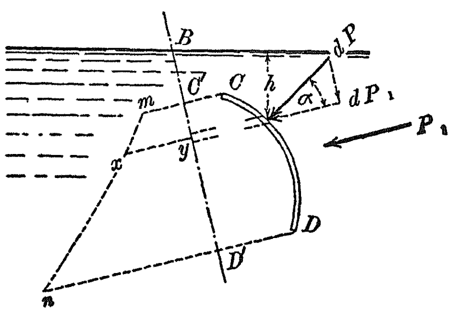

FIG. 19.

although the same reasoning and results would be obtained from the consideration of an irregular or double curved surface. Again, *C-D* will represent the edge view of our surface. We may imagine it to be such a surface as might be formed from a bent iron sheet. If we consider a small elementary area dA, the normal pressure upon it is

$$dP = dA \cdot wh, \text{ and } dP_1 = dP \cdot \cos\alpha = dAwh \cdot \cos\alpha.$$

Evidently the total value of P_1 may be expressed as

$$P_1 = \int dA \cdot wh \cos \alpha, \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where h and $\cos \alpha$ are both variable, having, *in general*, no relation. The expression cannot then be simplified, but a graphical representation of P_1 may be made. If on a plane perpendicular to P_1 the area C-D be projected, we obtain an area representing the integral of $dA \cdot \cos \alpha$. On each projection of a dA we may imagine a normal ordinate to be erected equal to the h for the corresponding dA in the real surface. Such an ordinate is x-y. A little thought will show that they do not increase in length in proportion to their distance from B (measured on C'-D') and, therefore, m-n is not a straight line. Between the plane C'-D' and the curved surface m-n (in which lie the ends of all the ordinates) is a volume representing $\int h\, dA \cos \alpha$, and if this be considered a solid, having a unit weight w equal to that of water, then its total weight will be $P_1 = \int wh\, dA \cos \alpha$.

The student will see that this procedure is wholly for illustrative purposes and has no practical application.

(3) **Special Cases.** — (*a*) *A curved or irregular surface with h assumed constant.* If in formula (12) the value of h be assumed constant, we have

$$P_1 = wh \int dA \cdot \cos \alpha = A_1 wh,$$

which is the same as formula (11) for plane surfaces. Often in practical work the head is so large that the above assumption can be made.

(*b*) Again, if the conditions are such that the angle α is constant, we obtain from (12)

$$P_1 = w \cos \alpha \int dA . h = w \cos \alpha A h_0$$

$$P_1 = A_1 w h_0 \text{ as before.}$$

Example. — As an illustration of this latter case, let it be desired to find the upward vertical component of normal pressure

on the submerged cone shown in Fig. 20. It is quite obvious that the angle α is constant for all portions of the elementary area, and we may proceed to find P_1 by projecting the conical surface upon a plane perpendicular to the direction of P_1 (*i.e.* a horizontal plane). The area thus obtained would be equal to that of the cone's base, or

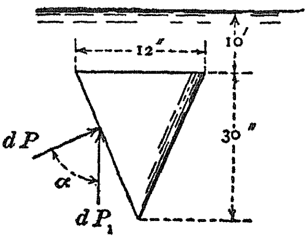

FIG. 20.

$$A_1 = .7854 \text{ sq. ft.}$$

$$\therefore P_1 = A_1 w h_0 = .7854 \times 62.5 \times 10\tfrac{10}{12}$$

or $P_1 = 531.8$ lb. *Ans.* 531.8.

It should be noted that h_0 is the head on the center of gravity of the *conical surface* and not the c. of g. of the cone's surface as *a whole*. For the former, the c. of g. is located $\frac{2}{3}$ of the distance from the vertex to the base.

(*c*) Perhaps most important of all to the engineer is the component of pressure in a horizontal direction; and it will now be proved that this may be found for *any kind* of surface.

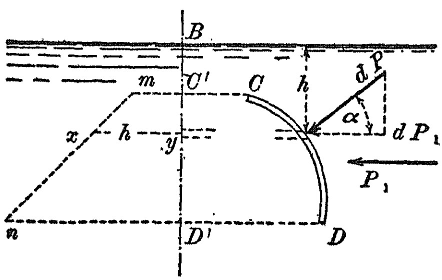

FIG. 21.

Referring to Fig. 21 and comparing it with Fig. 19, it will be seen that for this particular case each ordinate xy increases in direct proportion to its distance below the water surface, and the mean ordinate h_m will therefore be found at the center of gravity of the projected area, C'-D'. The volume formed from this area by the erection of ordinates on each dA will be $V = A_1 h_m$; and its weight, if of water, would be $W = A_1 w h_m$. It was pointed out, however, in Case 2 that this weight was the equivalent of P_1, so that we may write

$$P_h = A_1 w h_m, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

or,

The horizontal component of pressure on an area is the same as the normal pressure on the vertical projection of the area.

Example (Fig. 22). — Let it be required to find the magnitude of the resistance R which the bed of the stream must be able to offer the masonry dam in order to prevent its horizontal displacement by the pressure of the water. (Water 100 ft. deep.)

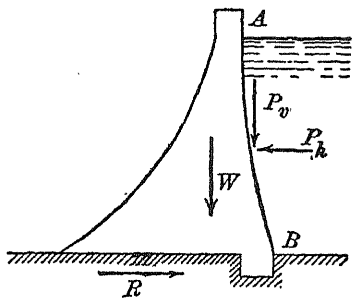

FIG. 22.

The forces acting on the dam are three in number; viz. its own weight W, the water pressure on A-B, and the reaction of the bed on the dam. (Of the latter force, R is the horizontal component.) Since these forces are in equilibrium, we may equate the algebraic sum of their horizontal components to zero, obtaining

$$P_h = R.$$

From (13)

$$P_h = (100 \times 1)\ 62.5 \times 50 = 312500 \text{ lb. per linear foot of dam.}$$

Note. — The general and special cases treated in this article should be thoroughly mastered by the student to avoid serious error in the solution of practical problems.

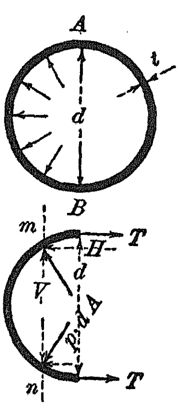

FIG. 23.

21. Pipes and Cylindrical Shells under Pressure (Fig. 23). — This is another practical illustration of the foregoing principles. It is desired to find the amount of tensile stress (hoop tension) in a pipe containing water under pressure. It will be assumed that in any cross-section A-B the intensity of pressure is the same at all points of the circumference. This is equivalent to assuming the head on all parts of the pipe the same. In Fig. 23 one half of pipe has been removed and the two tensile forces T

substituted to indicate the action of that half on the remaining portion. Any vertical line *m-n* passes through two elementary areas dA, on each of which the pressure is $p \cdot dA$. If we resolve these two pressures into horizontal and vertical components, we obtain two equal and opposite vertical forces, and two equal horizontal forces acting in the same direction. Evidently the sum of the H components acting on all the elementary areas of the semicircumference is equal to $2\,T$ (since for equilibrium $\Sigma H = 0$).

From Art. 20, the total H component of pressure is equal to the pressure on the vertical projection of the curved area, under the same head. If the length of pipe be unity, the area is d and $P_h = pd$. Finally, —

$$2\,T = P_h = pd.$$

$$T = \frac{pd}{2}.$$

This stress T is distributed over an area t, so that the stress per unit area is

$$f = \frac{pd}{2t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and

$$t = \frac{pd}{2f} = \frac{pr}{f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Ordinarily a pipe figured on the basis of this last formula would be too thin to withstand the strains arising from rough handling, unequal bearing, and water hammer, so that in practice it is customary to add an additional amount to the thickness obtained from (15).

Note. — Formula (15) could have been obtained without the assumption of constant head, since we were concerned with finding the *horizontal* component of normal pressure, and we have seen that this can be done without any assumptions (Art. 20). It is obvious, however, that with the pressure *varying* over the interior pipe surface, the two tensile forces T would not be equal and it would be necessary to assume them so before proceeding further.

22. Pressure on Both Sides of a Plane. — In the cases of immersed planes previously considered, the total pressure on one side equaled that upon the other, as the head on the c. of g. was unchanged. Imagine, however, a plane such as A-B (Fig. 24) separating the two water levels M and N, and let C-D be an edge view of a certain portion of its area. The total pressure on any small dA will be, for the left-hand side,

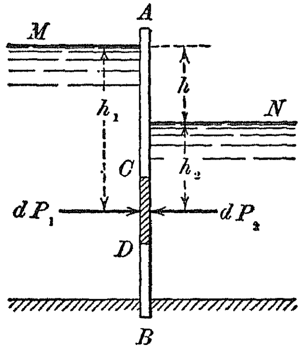

FIG. 24.

$$dP_1 = dA \cdot wh_1,$$

and for the right-hand side,

$$dP_2 = dA \cdot wh_2.$$

Their *resultant* (acting to the right) will be

$$dR = dP_1 - dP_2 = dA\ w(h_1 - h_2) = dA \cdot wh.$$

That is, the resultant pressure on any elementary area is dependent only on h, the difference in water levels. Each dA of C-D then has the same resultant pressure, and a single resultant applied at the c. of g. of C-D may replace these equal pressures. This is graphically shown in Fig. 25. The variation of pressure over A-B may be represented, on the left side, by the straight line O-E, and on the right side by A-F. The triangles O-B-E and A-B-F are similar, and A-B-F and C-D-E are equal. The trapezoid A-C-B-E represents the total pressure on the left of A-B, from which we must subtract the triangle A-B-F, or its equal, C-D-E, in order to obtain the resultant pressure. This will give the rectangle of pressure A-B-C-D, which shows the resultant pressure to be uniformly distributed over A-B.

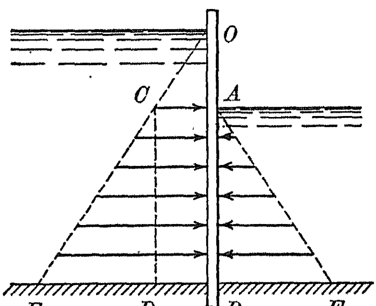

FIG. 25.

23. Pressure on Immersed Bodies. — *Theorem of Archimedes.* About the year 250 B.C. Archimedes discovered the important law that immersed bodies lose a portion of their weight equal to that of the volume of water displaced by them. The proof is simple (Fig. 26). AB is any such body through which we

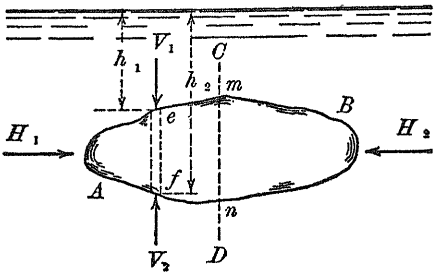

FIG. 26.

will pass a vertical plane C-D (perpendicular to plane of paper). The horizontal components of total pressure H_1 and H_2, on the irregular areas mAn and mBn, must be equal, as both are measured by the same projected area m-n (Art. 20). If the vertical plane C-D were passed through the body parallel to the plane of the paper, we should obtain $H_3 = H_4$, these latter forces being at right angles to H_1 and H_2. Evidently $\Sigma H = 0$ for all the normal pressures. To investigate the vertical components, assume a vertical prism e-f with end areas so small as to give uniform intensity of pressure upon them. By Art. 20, under these conditions we have

$$V_1 = dA \cdot wh_1,$$

dA being the area of the prism's cross-section. Similarly we obtain

$$V_2 = dA \cdot wh_2;$$

and the net resulting vertical force on the prism is

$$dR = V_2 - V_1 = dA \cdot w\,(h_2 - h_1).$$

This latter term is the weight of a volume of water equal to that of the prism. By a consideration of every elementary prism in the body, we may conclude that the body as a whole is acted upon by an upward resultant force equal to the weight of the volume of water displaced by the body.

If the body be floating at the surface with only a portion immersed, the law still holds good, as a consideration of the vertical pressures on an elementary prism will show. In either case we may conceive of two forces acting on the body,—the weight of the body and the *buoyant effort* of the water. The weight acts through the center of gravity of the body, while the buoyant effort acts through the center of gravity of the volume of displaced liquid. This may be seen by a return to an elementary prism. The small upward resultant force acting upon it being *proportional to its volume*, it follows that the final resultant of these elementary resultants must act through the center of gravity of the total volume. This point is called the *Center of Buoyancy*.

√**24. Depth of Flotation.**—To find the depth to which a floating body will sink, it is only necessary to remember that the weight of the body equals the weight of the displaced water. From the geometrical relations for any regularly shaped body we may then figure the distance.

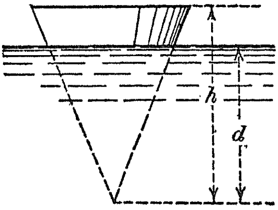

FIG. 27.

Illustration.—Pyramid or cone with axis vertical and vertex down (Fig. 27).

Let $V =$ volume of cone.
$w' =$ weight of cone per cubic unit.
$v' =$ volume of immersed part.

$$\text{Then} \quad \frac{v'}{V} = \frac{d^3}{h^3}, \text{ and } v' = \frac{d^3}{h^3} V.$$

The weight of the cone is $w'V$, and the weight of the water displaced is $w\frac{d^3}{h^3}V$, hence

$$w'V = w\frac{d^3}{h^3}V \text{ and } d = h\sqrt[3]{\frac{w'}{w}}.$$

The ratio $\frac{w'}{w}$ equals s, the specific gravity of the solid, so that finally,

$$d = h\sqrt[3]{s}.$$

Problem.—Let the student determine d for the cone or pyramid with axis vertical and vertex *up*. $d = h(1-\sqrt[3]{s})$.

25. Stability of Immersed and Floating Bodies. — If the weight of a body exceed the buoyant effort, it will sink. If it be less, it will, if placed beneath the surface, rise to the surface and assume a position according to the above stated principles. If the weight were just equal to the buoyant effort, it would remain beneath the surface wherever placed; or, if placed upon the surface, would sink until just submerged. Reference to Fig. 28 will show that for this latter case the body will assume a position so that its center of gravity and the center of buoyancy will lie in the same vertical line. This is necessary for equilibrium, as otherwise the weight at the center of gravity and the upward force at the center of buoyancy form a couple which will rotate the body. If the center of gravity is below the center of buoyancy, the couple will right the body and the equilibrium is stable. If the center of gravity be *above*, the condition is clearly that of unstable equilibrium; and if the two points coincide, the body is in equilibrium for all positions.

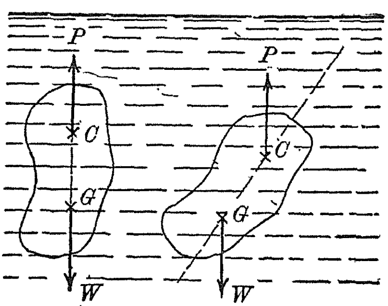

FIG. 28.

If the body *floats*, the conditions of equilibrium are not so easily determined. The position of the center of gravity will of course remain unchanged, but the volume of displaced water will alter its shape as the body is rotated, involving a change in the position of the center of buoyancy. This is well shown in the case of a floating ship (Fig. 29). Assuming the center of gravity to be at G and the center of buoyancy at C, or below G, it would seem at first sight as though the ship were in unstable equilibrium. However, consider an outside force to be applied causing the ship to roll (Fig. 30). The position of C is changed to some point C' lying on the side of greater immersion. In the figure shown, the line of action of the upward force cuts the axis A-B at a point M *above* the center of gravity. Evidently for this particular case the equilibrium is stable, as the couple tends to right the ship. If M had fallen *below* G,

the action of the couple would have tended to upset the ship, and the equilibrium would have been unstable. The point M is

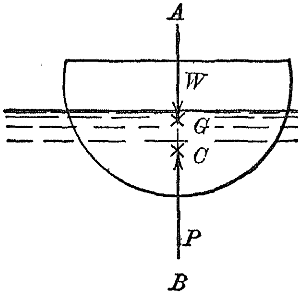

FIG. 29.

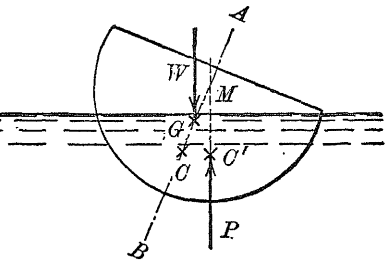

FIG. 30.

called the *Metacenter*. Its position relative to the center of gravity becomes then a criterion of stability, and it is important that we have some means for determining its exact location. As a matter of fact its position changes with different values of θ, the angle of displacement. As this angle becomes smaller and smaller, approaching zero as its limit, M approaches a limiting point which is the *true* metacenter, the position of M when $\theta =$ zero. It can be quite easily proved that the following equation will give the distance h_m measured from the center of gravity G (Fig. 30) to the metacenter M:

Let $V =$ volume of displaced water.

$I =$ least moment of inertia of a *water line* section through the body, about an axis through its own c. of g.

$c =$ distance between c. of g. and c. of b.

$= CG$ (Fig. 29).

Then
$$h_m = \frac{I}{V} \pm c. \qquad (16)$$

Here c is minus when the c. of g. is above c. of b.

For the derivation of this formula and a more detailed treatment of the subject the student is referred to an excellent article by Professor I. P. Church in his "Mechanics of Engineering," John Wiley & Sons, New York.

PROBLEMS

1. A rectangle is immersed with one side in the water surface. How must a straight line be drawn from one end of that side so as to divide the rectangle into two parts, the total pressures upon which shall be equal?

2. A gate 4 ft. wide and 6 ft. high, hinged at the upper edge, is kept closed by the pressure of water standing 8 ft. above the top. What force, applied normally at the bottom of the gate, would be required to open it?

3. A triangular area whose height is 12 ft. has its base horizontal and vertex uppermost in water; find the depth to which its vertex must be sunk so that the difference of level between the center of gravity and the center of pressure shall be 8 in.

4. Find the depth of the center of pressure on a trapezoid vertically immersed in water, the upper base being 5 ft. long, parallel to and 10 ft. below the water surface. The trapezoid is symmetrical about a vertical center line, its lower base being 3 ft. long and 13 ft. below the surface.

5. Find the center of pressure on a parabolic plate immersed to depth of 7 ft. with its axis of symmetry vertical.

6. A rectangular wrought-iron caisson is to be sunk, in which to build the foundation for a bridge pier. The caisson is in the form of a box without cover, 50 ft. by 20 ft. in plan. Weight of caisson is 75 tons; height, 23 ft.; depth of water, 20 ft. How deep will the caisson sink when launched? What load will sink it to the bottom?

7. A 5000-ton ship, drawing 25 ft., has to discharge 300 tons of water ballast to cross a 24-ft. bar into a river. She burns 50 tons of coal in going up the river into fresh water. How much is her draft then, and how much ballast will be needed to increase it by one foot?

8. An isosceles triangle, base 10 ft. and altitude 20 ft., is immersed vertically in water with its axis of symmetry horizontal. If the head on its vertex be 30 ft., locate the center of pressure both laterally and horizontally.

9. A wood stave pipe, 48 in. in diameter, is to resist a water pressure of 150 lb. per square inch. If the staves are held by flat steel bands, 4 in. wide by $\frac{3}{4}$ in. thick, find the spacing distance of the latter in order that they may not be stressed above 15000 lb. per square inch.

10. A gasholder contains illuminating gas under a pressure corresponding to 2 in. of water. If the holder be at sea level (atmospheric pressure 14.7 lb. per square inch), what pressure in inches of water may be expected in one of its distributing pipes at a point 500 ft. above sea level? Assume unit weights of air and gas to be constant at 0.08 and 0.04 lb. per cubic foot respectively.

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CHAPTER III

EFFECTS OF TRANSLATION AND ROTATION UPON BODIES OF WATER

26. Body of Water in Straight Line Motion. — (*a*) *Direction Horizontal.* Under this heading will be presented the single case of a mass of water having a motion of pure translation, but with its individual particles in a state of relative rest. Such a case may be illustrated by Fig. 31, in which is shown a reservoir partially filled with water and moving horizontally to the right. If the motion be *uniform*, then the forces acting on any particle are balanced and conditions are identical with those existing when the body is at rest. That is to say, the free surface will be horizontal and the pressure at any point in the mass follows the static laws already explained. Let us consider, however, that the reservoir has a uniformly *accelerated* motion toward the right, a being the value of the acceleration. If the reservoir starts from a state of rest, the surface bc will be observed to at first oscillate and then come to rest, occupying some such position as shown in the figure, making an angle α with the horizontal which we may determine as follows. Any particle in the surface must experience a single resultant force F, acting horizontally to the right, since its motion is uniformly accelerated and in that direction *only*. The measure of this force we know to be

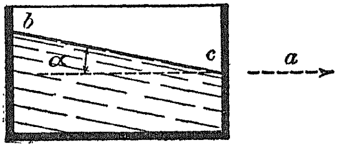


FIG. 31.

$$F = Ma.$$

Since upon each particle there is acting the force of gravity W, there must be present another force P which, when combined with W, gives F as a resultant. The parallelogram of

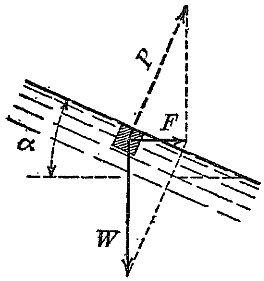


FIG. 32.

forces (Fig. 32) shows that P must act obliquely upward. It represents a force exerted by the surrounding particles, and if the particle considered is to remain at rest relative to its neighbors, this force must act normal to the free surface. The angle between P and the vertical is the angle α which the surface bc makes with the horizontal. The value of F as obtained from the figure is

$$F = W \tan \alpha,$$

and already we have

$$F = Ma = \frac{W}{g} a.$$

Eliminating W and F, we finally obtain

$$\tan \alpha = \frac{a}{g}. \quad \ldots \ldots \quad (17)$$

A little thought will show that this same result would follow, and bc occupy the same position, had the reservoir been moving to the *left* with motion uniformly *retarded*.

(*b*) *Direction Vertical*. If the reservoir be moving in a vertical path (downward or upward) with *uniform* velocity, the surface bc will remain horizontal, and the pressure throughout the water will be the same as if it were at rest. If the motion be uniformly accelerated, the surface will still remain horizontal, but the pressures in all parts of the water will change. Let the direction of motion be upward and the acceleration be positive in the same direction. Any elementary prism, as mn, we may treat as a rigid body in motion under the action of the forces shown (Fig. 33). Its weight is $dA \cdot w \cdot h$, and on its base is the total presure $p \cdot dA$. The horizontal forces (side pressures) are omitted, as they pro-

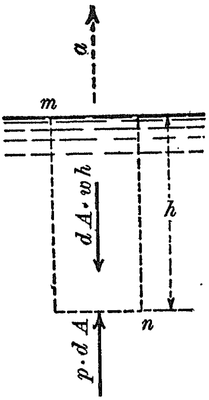


FIG. 33.

duce no vertical motion. The resultant force F equals mass $\times$ acceleration, or

$$p \cdot dA - dA \cdot w \cdot h = \frac{dA \cdot w \cdot h}{g} \times a,$$

from which

$$p = wh\left(\frac{g+a}{g}\right). \quad \cdots \cdots \quad (18)$$

For the special case of $a = g$,

we have $p = 2\,wh.$

If the acceleration be negative (*i.e.* mass moves upward with decreasing velocity), then a in (18) becomes negative. Thus, a mass of water moving upward, but coming to rest under the action of gravity alone, will have a pressure in all parts of

$$p = wh\left(\frac{g-g}{g}\right) = 0.$$

If the direction of motion now be changed to *downward*, with the acceleration positive in the *same direction*, a reconsideration of the forces acting on the elementary prism will show that

$$p = wh\left(\frac{g-a}{g}\right). \quad \cdots \cdots \quad (19)$$

If the body of water falls freely,

$$p = wh\left(\frac{g-g}{g}\right) = 0.$$

If the acceleration be in a direction opposed to motion, the negative sign in (19) becomes positive.

A little thought will make it clear that the law of pressure for a body moving downward with increasing velocity is the same as though moving upward with decreasing velocity; and for a body moving downward with decreasing velocity, the same as though moving upward with velocity increasing.

27. Body of Water in Rotation.— If a cylindrical vessel be partly filled with water and made to rotate with uniform velocity about its central axis, the contained water gradually ac-

quires the angular velocity of the vessel, and the free surface, at first horizontal, becomes dished or concave in form (Fig. 34). The mathematical nature of this surface and the reason for its formation may be understood if we consider the forces acting upon the fluid particles forming it. Selecting a small mass, located a distance x from the axis of rotation, let us designate its weight as ΔW. The only other force acting upon it is the pressure P it receives from the surrounding particles, and since it has no motion relative to the latter,

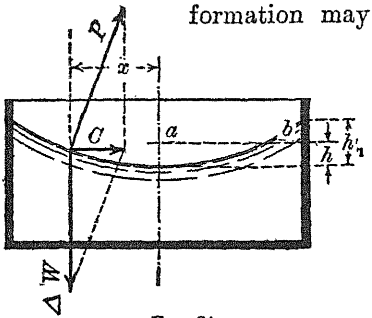


FIG. 34.

the direction of the pressure must be normal to the curved surface. The resultant of these two forces is the horizontal, deviating, or centripetal force C, whose magnitude is $\Delta M v^2 \div x$, v representing the linear velocity of the particle. If α represents the angular velocity of the vessel and the particle, v may be expressed as αx, giving

$$C = \frac{\Delta M \alpha^2 x^2}{x} = \frac{\Delta W}{g}\alpha^2 x.$$

Referring to Fig. 35, the tangent of the angle θ which P makes with the vertical is

$$\tan\theta = \frac{\Delta W \alpha^2 x}{\Delta W g} = \frac{\alpha^2 x}{g}.$$

But from the figure, θ is also the angle which a tangent to the curved surface at a makes with the axis of X. The tangent of θ may therefore be written

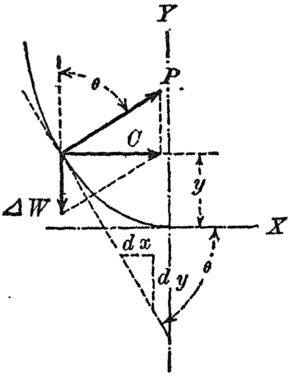


FIG. 35.

$$\tan\theta = \frac{dy}{dx} = \frac{\alpha^2 x}{g},$$

from which,

$$dy = \frac{\alpha^2 x}{g} dx,$$

and finally,

$$y = \frac{\alpha^2 x^2}{2g}. \qquad (20)$$

The equation shows the curve to be a parabola with vertex on the axis of rotation, and the form of the free surface is therefore that of a paraboloid of revolution. Its relation to the original surface is easily established, remembering that the volume of a paraboloid is one half that of the circumscribing cylinder. Thus in Fig. 34, if ab be the original level of the water and A the horizontal cross-sectional area of the vessel, we have

$$\text{Vol. paraboloid} = Ah_1 \div 2.$$

But, since the volume of water in the entire vessel remains constant,

$$Ah = Ah_1 \div 2,$$

and $$h = \frac{h_1}{2},$$

or we may say that the distance between the vertex of the paraboloid and the original level is the same as from the original level to the highest point on the new surface.

If the vessel had been entirely filled with water but closed at the top, the free surface described could not be formed; but the pressures throughout the liquid would be the same as though it were formed and had its vertex at the top of the vessel.

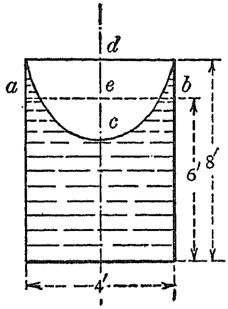


FIG. 36.

Example. — A cylindrical vessel (Fig. 36) is filled 6 ft. deep with water. Its height being 8 ft., and radius 2 ft., determine the angular velocity which will raise the water even with the brim. Find also the total pressure on the sides of the vessel.

By Art. 27, $ce = ed = 2$ ft.

$$\therefore\ cd = 4 \text{ ft.} = \frac{\alpha^2 x^2}{2g} = \frac{\alpha^2 4}{64.4}.$$

$$\alpha = 8.03 \text{ radians per second.} \quad \textit{Ans.}$$

(NOTE. — A radian is the angle of which 3.1416+ make a semicircumference.)

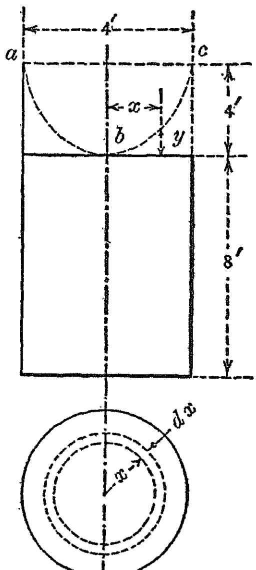
To find total side pressure: —

$$P = Awh_0 = (\pi \times 4 \times 8)\, 62.5 \times 4 = 25100 \text{ lb.} \quad \textit{Ans.}$$

Example. — If the vessel in the previous problem be full of water and closed at the top, determine the pressure on the top, bottom, and sides. $(\alpha = 8.03.)$

The closed top here prevents the formation of the parabolic surface, but inasmuch as the forces tending to produce it are present, the *pressure is the same* at any point as though it *were* formed. In this case the surface is virtually at *abc* (Fig. 37).

FIG. 37.



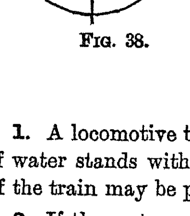


FIG. 38.

Pressure on sides, —

$$P = Awh_0 = (\pi \times 4 \times 8)\, 62.5 \times 8$$
$$= 50200 \text{ lb.} \quad \textit{Ans.}$$

Pressure on top, —

On an elementary ring (Fig. 38),

$$dP = dA\ wh = (2\,\pi x \cdot dx)\, wy,$$

y having the value $\dfrac{\alpha^2 x^2}{2g}$.

$$\therefore\ P = \frac{\pi\alpha^2 w}{g}\int_0^2 x^3 dx = 1570 \text{ lb.} \quad \textit{Ans.}$$

Pressure on the base, —

$$P = Awh + 1570$$
$$= (\pi 4 \times 62.5 \times 8) + 1570 = 7850 \text{ lb.}$$

Ans.

PROBLEMS

1. A locomotive tender contains a water tank 6 ft. square and 4 ft. deep. If water stands within 1 ft. of its top, what maximum uniform acceleration of the train may be permitted without losing water from the tank?

2. If the water which just fills a hemispherical bowl of 3 ft. radius be made to rotate uniformly about the vertical axis of the bowl at the rate of 30 revolutions per minute, how much water will overflow?

CHAPTER IV

FLUID MOTION — BERNOULLI'S THEOREM

28. Water in motion presents problems more difficult and uncertain in their solution than those encountered in Hydrostatics, because of the presence of frictional resistances and other disturbances whose actions often defy mathematical expression. As a result, we are in many instances obliged to leave these forces out of our analysis and derive formulæ expressing laws of flow which are far from being satisfactory if viewed from the standpoint of the Analytical Mechanics, depending as they do upon carefully performed experiments to furnish constants or coefficients to make good the discrepancies which would otherwise exist between results indicated by theoretically deduced formulæ and those obtained from experiment. The nature of the difficulties which arise will be seen later.

29. Fluid Motion. — Sinuous and Non-sinuous Flow. — It is a fact well established by experiment that water in motion along a channel may flow in either of two widely different ways, depending on the conditions present. Professor Osborne Reynolds of England in 1883 experimented on the flow of water through glass tubes and showed that at low velocities the water particles passed through the tube along straight parallel paths. This motion continued under gradually increased velocities until a velocity was reached at which it was noticed that portions of the tube remote from the entrance contained small eddyings. The presence of these eddies was noted by watching the progress through the tube of a fine stream of dye-colored water which had been injected into its entrance end. As the velocity was further increased the eddies became more numerous and finally filled all portions of the tube. The motion of the particles under these conditions was most complex and intricate. The velocity, at which the change of motion occurred, Reynolds

called the *Critical Velocity* and concluded from his experiments that its value for any particular pipe varied only with the temperature of the water, and in different pipes varied with the diameter if the temperature were constant. Further discussion of the Critical Velocity will be given in a later chapter, and it is enough to point out at this time the fact that water may have two different modes of motion and obviously two different laws governing the resistance to motion in each case.

30. Assumption of Steady, Non-sinuous Flow. — If a plane section be taken in a stream at right angles to the direction of motion, the flow is said to be "*Steady*" when the form of the cross-section and the conditions of velocity, pressure, and density remain constant. Each particle arriving at the section assumes the velocity and pressure which the preceding particle had, and although from section to section its velocity and pressure may change, at any one section these are constant in value. Such conditions of flow are generally present in all the problems with which the engineer has to do; and to problems of steady flow we will limit our discussion. It will be assumed that the motion of the particles is non-sinuous, or parallel, as by so doing we shall greatly simplify our mathematical work. Further, we will regard all particles in any cross-section of the stream to move forward with the same velocity, so that at successive sections the same particles will always be found together. In reality this is not so, as the presence of the channel wall retards those particles near it, causing, in general, the particles nearer the center of the section to have the fastest velocities.

31. Bernoulli's Theorem. — In 1738 Daniel Bernoulli demonstrated a general theorem in connection with water in motion, which perhaps is the most important in the subject of Hydraulics. Upon it as a framework may be hung the whole fabric of Hydrodynamics, and by its use alone a great majority of the problems arising may be completely solved. The assumptions of flow referred to in the preceding paragraph will be made, and the water regarded as a perfect fluid void of all friction. Figure 39 shows a portion of a stream passing through a pipe. The quantity Q passing any section in a given time must be

constant, else there will be an accumulation of water. Let v_1 and a_1 represent respectively the velocity and cross-sectional area at A, and v_2 and a_2 be the same quantities for the section at B. Then from the above,

$$a_1v_1 = a_2v_2 = Q.$$

This is often referred to as the "*Equation of continuity.*"

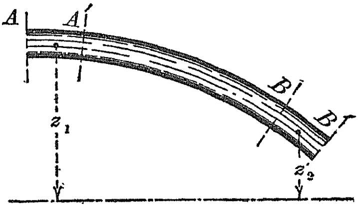


FIG. 39.

Consider now the body of water contained between these sections, and the forces acting upon it. The only working forces (those which produce motion) are the end pressures at the sections, and gravity. The pressures on the water due to the pipe are normal to the direction of motion, and perform no work. Let p_1 be the intensity of pressure at A, and p_2 that at B. Then the three working forces are a_1p_1, a_2p_2, and W. If they are allowed to act for a short interval of time, the work performed by them will result in a change in the kinetic energy of the water, and an equation may be written between the work

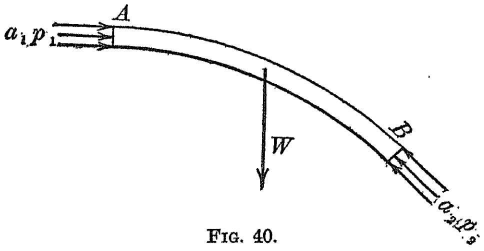


FIG. 40.

done and the corresponding change. Suppose that, in the time dt, A-B moves to A'-B' (Fig. 39). Then A-$A' = v_1dt$ and B-$B' = v_2dt$.

$$\text{Work done by } a_1p_1 = +a_1p_1 \cdot v_1dt = p_1Qdt.$$
$$\text{Work done by } a_2p_2 = -a_2p_2 \cdot v_2dt = -p_2Qdt.$$

This last is negative, since it opposes motion. As for the work done by gravity during the move from A-B to A'-B', we may consider the change in the water's position to have been accomplished by moving A-A' to B-B', and the work done by gravity would be that of making such a change.

∴ Work done by gravity $= wa_1 \cdot v_1 dt\,(z_1 - z_2) = wQdt\,(z_1 - z_2)$, where z_1 and z_2 represent the heights, above any assumed datum plane, of the centers of gravity of A-A' and B-B'.

We have now to find the change in kinetic energy of A-B. Conceive for a moment that all the particles of water between A and B be divided into a great number of groups of equal volume, the volume of each being that of A-A', which is the first group. Some of these are shown in Fig. 41, greatly exaggerated in size. With this assumption it is easily seen that as group a moves forward to displace b, in the time dt, its particles gradually assume the velocities that those in b had at the beginning of the time dt, and thus the group experiences a change in kinetic energy. Simultaneously with this movement, each group from a to m advances into the position previously held by the immediately preceding group, and m moves to B-B' at n. The change in energy experienced by any group, as c, would be the difference between the original kinetic energies of c and d, inasmuch as c acquires d's velocity and (having the same mass) its kinetic energy. The total change in kinetic energy for the entire body A-B would be obtained therefore by adding the differences between contiguous groups. There results : —

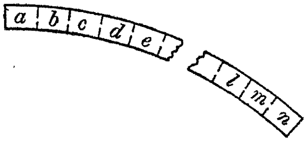


FIG. 41.

Total change in $K=$

$$(k_b - k_a) + (k_c - k_b) + (k_d - k_c) + \text{etc.} \ldots (k_n - k_m),$$

or, $$\text{Change in } K = k_n - k_a.$$

The change is then equal to the difference between the kinetic energy of A-A' and B-B', or,

$$\text{Change in } K = \frac{w \cdot Q \cdot dt}{2g}(v_2^2 - v_1^2).$$

Finally, equating work done to change in energy we have

$$p_1 Qdt - p_2 Qdt + wQdt\,(z_1 - z_2) = \frac{wQdt}{2g}(v_2^2 - v_1^2), \quad (21)$$

or, $$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2. \quad . \quad . \quad . \quad . \quad (22)$$

This expression constitutes Bernoulli's Theorem. It states that under steady flow, with friction eliminated, the sum of

$$\frac{v^2}{2g} + \frac{p}{w} + z = \text{a constant quantity} \quad . \quad . \quad . \quad (23)$$

for all sections.

Each of these three terms is a linear distance. The value $\frac{p}{w}$ we are already familiar with, it being the head that gives rise to the pressure p, and known as *Pressure Head*. The term z is simply the height of the particle above any assumed datum and will be known as the *Potential Head*. Lastly, $v^2 \div 2g$ is *Velocity Head*. The significance of the name will appear shortly. The sum of these three heads for any particle we will call the *Total Head*.

Our last equation may now be thus expressed in words: *In steady flow, without friction, the sum of velocity head, pressure head, and potential head is a constant quantity for any particle throughout its course.*

32. Bernoulli's Theorem with Friction. — It will be of interest to note here the effect on Bernoulli's Theorem of introducing friction. The presence of friction produces resistances or forces which *hinder* motion. The work of such forces is therefore negative, and equation (21) must be changed by adding to the left-hand member a negative quantity to represent this work. The quantity must contain a force and a distance, and we may employ any force we please provided a proper distance be used. Selecting the force $w \cdot Qdt$, which already appears in the equation, and representing the distance by h', we have

$$p_1 Qdt - p_2 Qdt + wQdt(z_1 - z_2) - wQdt\, h' = \frac{wQdt}{2g}(v_2^2 - v_1^2),$$

or
$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 - h' = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2, \quad . \quad . \quad . \quad (24)$$

or
$$h' = \left(\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1\right) - \left(\frac{v_2^2}{2g} + \frac{p_2}{w} + z_2\right). \quad . \quad . \quad (25)$$

Here in (25) h' is seen to be the difference between the totals of the heads at the two sections. We may then call it the

Lost Head due to friction (sometimes spoken of as "Friction Head"). Again, equation (25) may be thus expressed in words: *In steady flow, with friction, the total head at any section is equal to that at any subsequent section plus the lost head occurring between the two sections.*

33. Losses in Head. — The study of causes or conditions leading to loss in head and the methods for calculating the latter form a considerable portion of the hydraulics; and although full discussion of them will be given in later chapters, mention will be made here of three ways in which the losses may occur.

(1) Loss due to the internal friction existing between individual water particles. Since the water is not a perfect fluid, but slightly viscous, the motion of the particles past each other causes them to lose energy (and therefore head) in performing useless work.

(2) Loss due to friction between the moving water and the surface which surrounds or contains it. This is the most important loss, as regards magnitude, if the extent of the rubbing surface be considerable; and in many problems it becomes so large as to allow all other sources of loss to be neglected.

(3) Loss due to the sudden reduction of the velocity of the moving water, as when an abrupt enlargement in the pipe or channel is encountered. This loss may be divided into two parts, one due to energy lost in impact, and the other caused by the violent eddyings of the water and the consequent internal friction.

34. Application of Bernoulli's Theorem. — In a majority of the problems arising in practice, it is the velocity or pressure at a certain point that is desired. In experimental work more often it is the value of the lost head that is unknown and sought. Since there are seven heads appearing in the equation, and only one, in general, can be unknown, it is necessary in choosing the points or sections between which we write the theorem, to find points at which all the heads, save the one desired, are known. In most cases, as we shall see later, this is possible. Often where there appear to be two unknowns, a second equation may be written giving a relation between

them. Thus if one velocity head be unknown, it is often possible to express it in terms of the velocity of another point from the relation

$$a_1 v_1 = a_2 v_2 = a_3 v_3, \text{ etc.}$$

Later we shall see that the values of head lost under any circumstance may be expressed as a function of the velocity, so in reality only *six* heads appear in the general equation.

The student is early advised to acquire the power of applying this important theorem with confidence and understanding, as it will prove to be the key to many an otherwise difficult or lengthy problem.

Example. — A pipe line gradually enlarges from 24 in. in diameter at A to 36 in. at B. The velocity at A is 5 ft. per second, and the pressure, above that of the atmosphere, 50 lb. per square inch. Assuming that 2 ft. of head are lost between A and B, find the pressure at the latter point if it be situated 15 ft. lower in elevation than A.

From Bernoulli's Theorem,

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2 + \text{Lost head.}$$

From the conditions stated,

$$v_1 = 5 \text{ ft. per second.}$$

$$p_1 = 50 \text{ lb. per square inch.}$$

$$z_1 = 15 \text{ ft.}$$

$$z_2 = 0 \text{ ft.}$$

$$\text{Lost head} = 2 \text{ ft.}$$

(It will be noted that the datum from which the z's were reckoned was assumed passing through the point B.) As for the value of v_2, we have

$$a_1 v_1 = a_2 v_2,$$

giving

$$v_2 = \frac{a_1 v_1}{a_2} = \left(\frac{24}{36}\right)^2 \times 5 = 2.22 \text{ ft. per second.}$$

Substitution of the above values gives

$$\frac{25}{64.4}+\frac{50\times 144}{62.5}+15=\frac{4.93}{64.4}+\frac{p}{w}+0+2.$$

$$\frac{p}{w}=128.4 \text{ ft.}$$

$p=8025$ lb. per square foot, or 55.7 lb. per square inch. *Ans.*

35. Pressure can never be Negative.—Inspection of Bernoulli's Theorem, written without friction head,

$$\frac{v^2}{2g}+\frac{p}{w}+z=\text{Constant head,}$$

shows that if v be increased at any point and z kept at a constant value, the value of the pressure head must be decreased. It would seem, therefore, that by causing v to increase without definite limit we might reduce the pressure not only to zero, but cause it to have a negative value. A little thought will show that a condition of negative pressure is impossible, as water cannot stand tension (save in so slight a degree as to be negligible). If in the solution of the general equation a negative value of p be obtained, it may at once be known that either an error has been made or that flow does not take place under the conditions assumed. If a tendency to cause negative pressure does exist, the stream must break apart at such a point, and the tube or channel will be filled with water vapor under tension.

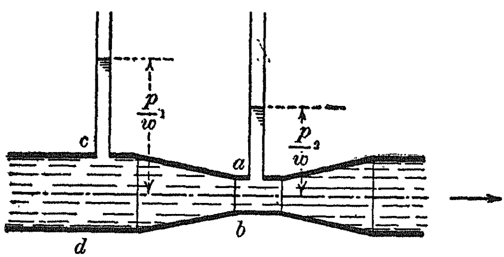


FIG. 42.

It is also worth noting here, that, since by increasing the velocity head a diminution in pressure may be brought about, the pressure at a contracted section of a pipe (like *a-b*, Fig. 42) is less than at the full section *c-d*. This is contrary to

the popular notion that the pressure is *increased* by reason of the water being "*squeezed through*" the smaller section.

36. Piezometer Measurements.—In Fig. 42 are shown two open piezometer tubes inserted in the pipe, and the heights of their water columns are marked $\frac{p_1}{w}$ and $\frac{p_2}{w}$, indicating that they measure the true pressure heads at the two points. That this is so may be seen from the fact that the *water in the columns is at rest*, and hence their heights must be an exact measure of the pressure at their bases.* The piezometers and gauges described in Arts. 12, 13, and 14 may therefore be used for determining pressures or differences in pressure existing in pipes containing flowing water. It is important, however, that in their use certain precautions be taken regarding the manner of inserting them into the pipe wall. Figure 43 shows three tubes, of which (a) is normal to, and has its inner edge flush with, the surface of the pipe wall. It measures the pressure at its base correctly. Both (b) and (c) are inclined to the inner surface, although their ends are flush with it. The water in (b) will stand lower than in (a), while in (c) it will stand higher. Both give incorrect readings on account of their columns being affected by the velocity of the moving water. Had their lower ends not been flush with the inner surface, the discrepancies would have been further magnified. In the case of the tube (a) it has been supposed that cohesion might cause some of the water to be drawn out of the tube and the level to be lowered. Careful experiments have shown that such is not the case if care be taken to make the tube normal to, and flush with, the surface.

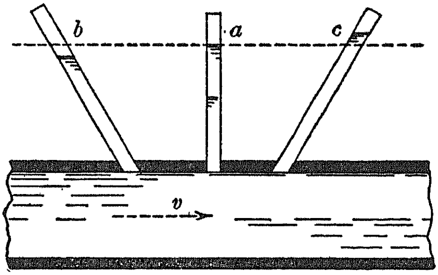


FIG. 43.

* The pressure at a point in the moving stream directly beneath the column is also indicated by the column's height above the point, since the prism of water below the column has no *vertical* acceleration.

37. Energy of Water in Motion.— A body moving through space, unhindered by friction, has a constant stock of energy which exists partly as *potential* and partly as *kinetic* energy. Thus a body may start from a state of rest with only potential energy, and, having fallen through a height h, acquire a kinetic energy which is the exact equal of the potential energy lost in that distance. If at this instant it be a distance z above some horizontal plane of reference, its total stock of energy must be

$$E = Wz + \frac{Mv^2}{2} = W\left(z + \frac{v^2}{2g}\right),$$

or, *per pound of substance,*

$$E = z + \frac{v^2}{2g}.$$

With a mass of moving water (friction eliminated) it is also true that its stock of potential and kinetic energy is constant, if we consider the *entire mass* concerned. If, however, we consider a small portion of it only, we find that such is not the case. Thus in a pipe of constant diameter it will be seen that, if the pipe is not horizontal, a small mass may be constantly losing potential energy as it progresses and yet receive no addition to its kinetic energy (velocity the same at all points). If its total stock of energy is to remain constant (friction eliminated), we must recognize the existence of another form of energy which we may, for want of a better name, call *pressure energy*. This we will define as the ability of the water to do work by virtue of the pressure derived from its contact with adjacent masses. That this ability does exist may be shown by discussing the action of a very simple water motor.* We will imagine a closed horizontal cylinder of small cross-section and length l, fitted with a piston and communicating by means of properly arranged valves with a large reservoir of water. The piston being at one end of its stroke, the opening of a valve admits water under a pressure p, and the piston moves to the other end of the cylinder. The valve being now closed and an exhaust port opened, the contained water is driven from the cylinder without resistance by a return move-

* Hydraulics, p. 717, I. P. Church.

ment of the piston now receiving water on its other face from the opposite valve. The work of a single stroke is $A \cdot p \cdot l$, and the weight of the water used is $A \cdot l \cdot w$. We may then say that, *per pound of water used*,

$$\text{Work done} = \frac{Apl}{Alw} = \frac{p}{w} \text{ ft. lb.}$$

While in the motor, the water gave up none of its potential energy (motion horizontal), neither did it give up kinetic energy, since its velocity at the beginning and at the end of the stroke was zero. The work done by the water, therefore, was at the expense of its pressure, which fell from p to zero (relative pressures) as it passed from the cylinder. It should be noted that the water used depended for its energy on its connection with a mass of water having a higher potential; and had it not been for the presence of the latter, no work could have been done. Pressure energy may be said to be the existence of potential energy in another form, and in the case of the small mass of water contained in the pipe mentioned at the beginning of this article, a loss in potential is balanced by a gain in pressure energy. We may write, for any particular position of the mass,

$$\frac{v^2}{2g} + \frac{p}{w} + z = \text{a constant}, \quad . \quad . \quad . \quad . \quad (26)$$

the three terms representing, respectively, the kinetic, pressure, and potential energy of the mass *per pound of weight*. We recognize in this equation Bernoulli's Theorem, and see that the latter is really an expression of the principle of Conservation of Energy.

If the movement of the water is accompanied by friction, the sum of the three energies in (26) is not constant and between any two points in the stream we should have

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2 + \text{Lost energy}. \quad . \quad (27)$$

The energy per pound of water passing a section being given by (26), the total energy of the stream past that section would

be obtained by multiplying by W, the weight of water passing the section.

PROBLEMS

1. Water is supplied to a locomotive tender by means of a scoop dipping into a track trough. If the point of entrance into the tank is 7 ft. above the trough level, and 5 ft. of head be lost in friction as the water passes up the trough, determine the velocity of flow into the tank when the engine is running 40 miles an hour. What is the minimum speed at which water can be delivered to the tank, assuming the friction head as zero?

2. Water is to be pumped over a hill through a pipe 3 ft. in diameter, so that at the top of the hill (elevation 300 ft. above pumps) there shall be a pressure of thirty lb. per square inch. The quantity of water is 49.5 cu. ft. per second. If a head of 10 ft. is lost between the pumps and the summit, what must be the theoretical horse power of the pumps?

3. Water enters a motor through a 4-in. pipe under a gauge pressure of 150 lb. per square inch. It leaves by an 8-in. pipe at an elevation 3 ft. below the point of entrance. If the pressure in the pipe at exit be 10 lb. per square inch and the discharge 2 cu. ft. per second, find the work done on the motor in one second by the water, assuming no loss by friction.

4. How many horse powers are being transmitted through a 3-in. pipe in which the velocity of flow is 15 ft. per second and the accompanying gauge pressure 40 lb. per square inch?

5. During the test of a centrifugal pump, a gauge just outside the casing and on the 8-in. suction pipe registered a pressure 4 lb. per. square inch less than atmospheric. On the 6-in. discharge pipe another gauge indicated a pressure of 30 lb. per square inch. If a vertical distance of 3 ft. intervened between the pipe centers at the sections where the gauges were attached, what H. P. was expended by the pump in useful work when pumping 2 cu. ft. per second?

6. Water flows radially outward in all directions from between two horizontal circular plates which are 4 ft. in diameter and placed 1 in. apart. A supply of 1 cu. ft. per second being maintained by a pipe entering one of the plates at its center, what pressure will exist between the plates at a point 6 in. from the center if no loss by friction takes place?

CHAPTER V

DISCHARGE FROM ORIFICES

38. Orifices with Sharp Edges. — Water flowing through an orifice having sharp edges presents the following characteristics. As it leaves the orifice it gradually contracts to form a jet whose cross-sectional area is somewhat less than that of the orifice (Fig. 44). This is due to the motion the particles have along the wall of the containing vessel as they approach the orifice. The contraction is not complete until the plane *a-b* is reached. For a circular orifice, having its edge in the same plane as the inner wall of the vessel, *a-b* is distant from that plane approximately one half the diameter of the orifice. At this point in the jet and at subsequent sections, the pressure throughout a cross-section is that due to the atmosphere, and in general this is true for any jet formed free in air. If a portion of the jet included between two normal sections *a-b* and *c-d* be considered, it will be seen that this mass is falling freely through the air without horizontal acceleration (resistance due to atmosphere neglected). The only force acting on it is that of gravity, and by Art. 26 it will be seen that the pressure in the mass must be atmospheric.

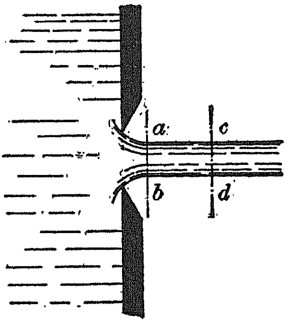


Fig. 44.

In the short portion of the jet preceding *a-b*, the pressure is greater than that due to atmosphere, since the particles are moving in curved paths, and experience a pressure due to centripetal force.

Let Fig. 45 represent a sharp-edged orifice in the side of a large tank or reservoir, having a depth of water, or head, on its center equal to h. Since this head is maintained constant

by the inflow at E, the case is one of steady flow. The area of the reservoir surface will be assumed to be very large compared with the area of the orifice, the result being that the particles in the surface will have no appreciable downward *Velocity of approach*. This being so, their velocity *head* may be taken as zero. Eliminating for the moment all friction, we will apply Bernoulli's Theorem to a particle in the surface A-B, and in its later position C, at the contracted section of the jet.

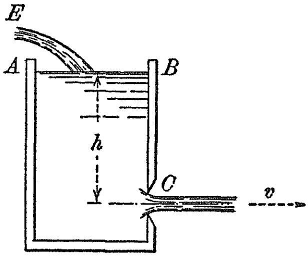


FIG. 45.

The pressure head when at the first position is that due to atmosphere, or $\frac{p_a}{w}$. Assuming the datum plane as passing through the orifice, the potential head will be h. At C the velocity head is $\frac{v^2}{2g}$, the pressure is again atmospheric, and the potential head zero.

There results,

$$0+\frac{p_a}{w}+h=\frac{v^2}{2g}+\frac{p_a}{w}+0,$$

or

$$v=\sqrt{2gh},$$

which is the theoretic value of the velocity, friction being eliminated. Several important deductions at once follow. We see that the water attains a velocity equal to that which it would have if it had fallen freely through the height h. In corroboration of this fact we would expect that, if the orifice were horizontal and the jet directed upward (Fig. 46), the stream would rise to a height equal to the head which produced it. Experiment verifies this, although for heads over six or eight feet the resistance by the atmosphere, and friction at the orifice, becomes great enough to cause a slight discrepancy. This is more marked as the head increases.

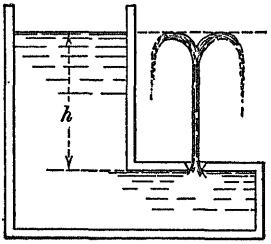


FIG. 46.

From the relation $h = \frac{v^2}{2g}$ it will now be seen why in Art. 31 we gave the name "velocity head" to the term $\frac{v^2}{2g}$, it being the head which would produce the velocity v. This relation holds good for all cases of steady flow *without friction*, as we shall see later.

39. Coefficient of Velocity. — It is found by experiment that the actual value of v for the jet is a little less than that found from $v = \sqrt{2gh}$, mostly because of friction at the edge of the orifice but also because water is not a perfect fluid. The loss in velocity is about two per cent, giving as a value for actual velocity,

$$\text{Actual vel.} = 0.98\sqrt{2gh} = c_v\sqrt{2gh}, \quad . \quad . \quad (28)$$

where .98 is called the *Coefficient of Velocity*. The latter varies but little in a range of several hundred feet of head, so we may use the value .98 in all numerical work.

It might be noticed that the theoretic value of v applies strictly only to orifices in a horizontal plane, all parts of the orifice being under the same head. With the orifice vertical and h measured to its center, the value of v obtained from $v = \sqrt{2gh}$ will not be a *mean* value for the cross-section of the jet, inasmuch as v varies with $\sqrt{h}$. The difference, however, is but a small fraction of one per cent, and is cared for by the coefficient.

Since v varies as $\sqrt{h}$, it will be seen that the curve in Fig. 47, showing this variation, is a parabola with vertex in the water surface. It is obvious that the variation in velocity throughout the cross-section of an orifice is greater as h decreases, and for very low heads the above statement regarding the magnitude of the variation is not true. We are at present assuming, however, that the head is large in proportion to the vertical dimension of the orifice.

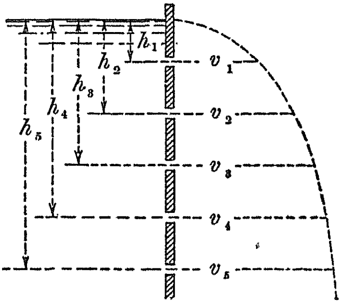


FIG. 47.

40. Coefficient of Contraction.— This may be stated as the ratio of the area of the contracted section to that of the orifice. It has been found to vary slightly with the size of the orifice and with the head, having a mean value of about 0.62.

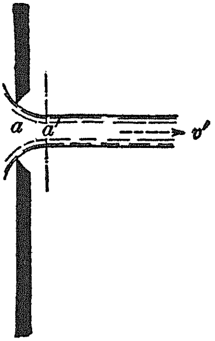


FIG. 48.

$$\therefore c_c = \frac{a'}{a} = 0.62 \text{ (Fig. 48).}$$

41. Coefficient of Discharge. — The quantity Q flowing from the orifice per second may be stated as the product of a' and the actual velocity past that section, and we may write

$$Q = a'v' = ac_cv' = ac_cc_v\sqrt{2gh},$$

or

$$Q = c_d a\sqrt{2gh}, \quad \ldots \ldots \quad (29)$$

where $a\sqrt{2gh}$ represents the theoretical discharge and c_d is the product of $c_c \times c_v$. We shall call it the *Coefficient of Discharge*, and its mean value will be given as $0.62 \times 0.98 = 0.61$. Since it varies with c_c and c_v, for strictly accurate work its value must be determined for each particular case.

42. Determination of the Value of the Coefficients. — (*a*) *Coefficient of Contraction.* The only direct way of measuring the amount of contraction is to caliper the contracted section. This may be done by inclosing the jet with a ring containing finely pointed screws, set radially in its circumference, which can be adjusted so as to just touch the jet as it passes the ring. The latter may then be removed and, if the screws be set on diameters, these may be measured and a mean diameter obtained. The method is not wholly satisfactory, as it is difficult to keep the jet steady. Another method consists in first determining the coefficients of velocity and discharge, and then using the relation

$$c_c = c_d \div c_v.$$

(*b*) *Coefficient of Velocity.* This is generally obtained by making a series of measurements on the path of the jet. If a

particle issues from the orifice with a velocity v, and in t seconds is found at the point m (Fig. 49), we may write

$$y = vt,$$

$$\text{and } x = \frac{gt^2}{2} \text{ (freely falling body).}$$

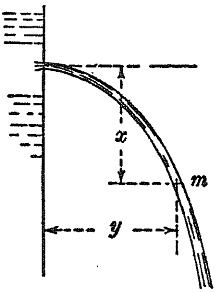


FIG. 49.

Between these two equations we may eliminate t, obtaining

$$y^2 = \frac{2\,v^2}{g}x,$$

which shows the path to be a parabola with vertex at the orifice. Evidently if we measure the x and y of any point in the jet, we may use the above equation to determine the actual velocity of flow from the orifice. The coefficient is then obtained from

$$c_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}.$$

The errors present in this mode of determination arise not only from inaccurate measurement of x and y, but from the fact that the atmosphere retards the particles after leaving the orifice. The values of x and y are obtained by placing a ring in a supporting frame, and adjusting the whole so as to cause the jet to pass through the ring. This done, the flow is stopped and the coördinates of the ring measured. This may be done at several different points and a mean value for v obtained. With the exercise of care, it is probable that a very good determination of the cofficient may be made.

(*c*) *Coefficient of Discharge.* This may be more easily found than the others, since it is only necessary to allow the orifice to discharge for a known length of time and measure (by volume or weight) the amount of water passed. The actual and theoretical quantities may then be compared for the coefficient.

The values of c_v and c_c are seldom required by the engineer, and it is only c_d (or simply c, as we shall hereafter call it) that is of interest to him. To determine this, thousands of experiments have been made in order that its value may be known

for a wide range of heads and sizes of orifice. Table II of the Appendix gives a short list of values as taken from a more extended table given by Hamilton Smith, Jr., in his "Hydraulics." It will be noticed that they decrease with an increase in head or diameter of orifice.

Example. — Required the velocity of flow and volume discharged per second from a circular orifice 0.2 ft. in diameter, under a head of 9 ft. Assume $c = 0.60$.

By equation (28),

$$v = 0.98\sqrt{2 \times 32.2 \times 9} = 23.6 \text{ ft. per second.}$$

By equation (29),

$$Q = 0.60 \times 0.03\sqrt{2 \times 32.2 \times 9} = 0.43 \text{ cu. ft. per second.}$$

43. Large Vertical Orifices under Low Heads. — So far we have dealt with an orifice whose vertical dimension has been small compared to the head upon it. With large orifices under low heads, the variation of velocity in the jet's cross-section gives rise to a discharge differing slightly from that obtained by use of formula (29). It will now be shown, however, that the difference is slight and may be neglected, provided the head is at least twice the vertical dimension of the orifice.

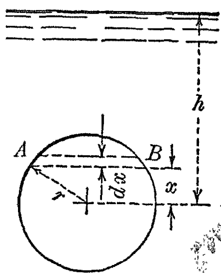


FIG. 50.

Case 1. *Circle* (Fig. 50). — As before, h will represent the head on the center of the orifice. If A-B be any elementary strip, drawn horizontally across the orifice at a distance x from its center, we have for a small discharge through it: —

$$dQ = dA \cdot v = 2\sqrt{r^2 - x^2} \cdot dx \times \sqrt{2g(h - x)}$$
$$= 2\sqrt{2g}(r^2 - x^2)^{\frac{1}{2}}(h - x)^{\frac{1}{2}}dx.$$

By making x vary between the values $-r$ and $+r$, the integration of this expression will give the discharge from the entire orifice. It will be necessary to expand the term $(h - x)^{\frac{1}{2}}$ by the binomial theorem.

$$(h-x)^{\frac{1}{2}} = h^{\frac{1}{2}} - \frac{h^{-\frac{1}{2}}x}{2} - \frac{h^{-\frac{3}{2}}x^2}{8} - \frac{h^{-\frac{5}{2}}x^3}{16} - \text{etc.} \cdots$$

$$\therefore\ dQ = 2\sqrt{2g}\left[(r^2-x^2)^{\frac{1}{2}}h^{\frac{1}{2}} - \frac{(r^2-x^2)^{\frac{1}{2}}x}{2h^{\frac{1}{2}}} - \frac{(r^2-x^2)^{\frac{1}{2}}x^2}{8h^{\frac{3}{2}}} - \frac{(r^2-x^2)^{\frac{1}{2}}x^3}{16h^{\frac{5}{2}}} - \text{etc.} \cdots\right] dx.$$

Each term of this is now possible of integration, and there results

$$Q = \pi r^2\sqrt{2gh}\ \left(1 - \frac{r^2}{32h^2} - \frac{5r^4}{1024h^4} - \frac{105r^6}{65537h^6} - \text{etc.}\right) \quad . \quad (30)$$

which is an exact theoretic formula for the discharge. An inspection of the parenthesis quantity shows it to have a value less than unity, and the discharge is therefore less than that given by the formula

$$Q = a\sqrt{2gh}$$

previously obtained for relatively large heads. This is what we might have anticipated.

Let us inquire into the numerical value of the parenthesis quantity as we assign different values to the ratio $\frac{h}{r}$. If $\frac{h}{r}=2$, or the head is just equal to the diameter, the quantity becomes 0.992. With $\frac{h}{r}=4$, or $h=2d$, the value is 0.998. The latter figure shows that with all ordinary ranges of head we may neglect the refinement of formula (30), and for figuring actual discharge use

$$Q = ca\sqrt{2gh},$$

the value of c being found in Table II.

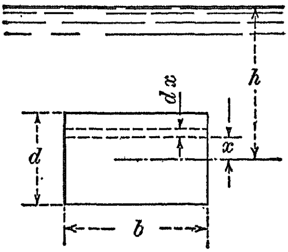


FIG. 51.

Case 2. *Rectangle* (Fig. 51). — In this case the small discharge dQ through an elementary strip parallel to the surface may be written, as before,

$$dQ = dA \cdot v = b \cdot dx \cdot \sqrt{2g(h-x)}$$

or

$$dQ = b\sqrt{2g}(h-x)^{\frac{1}{2}} \cdot dx$$

where the limits of x are $-\frac{d}{2}$ and $+\frac{d}{2}$. If we expand $(h-x)^{\frac{1}{2}}$ as before,

$$Q = b\sqrt{2gh}\int_{-\frac{d}{2}}^{\frac{d}{2}}\left(1-\frac{x}{2h}-\frac{x^2}{8h^2}-\frac{x^3}{16h^3}-\frac{5x^4}{128h^4}-\text{etc.}\cdots\right)dx,$$

or

$$Q = bd\sqrt{2gh}\left(1-\frac{d^2}{96h^2}-\frac{1}{2048}\frac{d^4}{h^4}-\text{etc.}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

As in the previous case, we see that the value of the parenthesis is less than unity. If $h = d$, its value becomes 0.989, while for $h = 2d$, it becomes 0.997. Then we may say that for heads greater than twice the depth of the orifice, we may figure the actual discharge from

$$Q = ca\sqrt{2gh}.$$

Values of c for *square* orifices are found in Table III. Table IV gives values of c for *rectangular* orifices 1 ft. wide and of varying depths. Both tables are taken from Hamilton Smith's "Hydraulics."

Some experiments by Ellis on a large orifice 2 ft. square under heads varying from 1.8 to 11.3 feet gave a coefficient nearly constant in value and equal to 0.60.

44. Recapitulation. — It is well to bear in mind the conditions under which we have so far studied orifices. We have assumed: —

(*a*) Ratio of reservoir surface to orifice area very large, *i.e.* *no velocity of approach.*

(*b*) Reservoir surface and jet both under *atmospheric* pressure.

(*c*) A *sharp-edged* orifice.

(*d*) No suppression of the contraction.

A departure from any one of these conditions will lead to material changes in the flow, the natures of which will be shown in the succeeding paragraphs.

45. Velocity of Approach. — Figure 52 shows, in longitudinal section, a channel bringing water to an orifice. Because of a

relatively small cross-section, there exists in the channel a velocity of approach which we will assume to have a value V at all points (not really so). Applying our theorem to the points m and n, we obtain

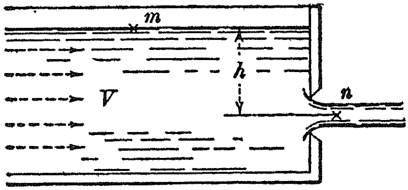


FIG. 52.

$$\frac{V^2}{2g} + \frac{p_a}{w} + h = \frac{v^2}{2g} + \frac{p_a}{w} + 0,$$

or
$$v = \sqrt{2gh + V^2}, \quad . \quad . \quad . \quad . \quad . \quad (32)$$

this being the *theoretic* velocity. Since flow is steady,

$$AV = cav, \text{ and } V = \frac{cav}{A},$$

in which A represents the area of the channel's cross-section.

Substituting in (32) and simplifying,

$$v = \sqrt{\frac{2gh}{1 - \left(\frac{ac}{A}\right)^2}}.$$

∴ for the actual velocity,

$$v = c_v \sqrt{\frac{2gh}{1 - \left(\frac{ac}{A}\right)^2}},$$

and
$$Q = c_c a c_v \sqrt{\frac{2gh}{1 - \left(\frac{ac}{A}\right)^2}} = a \sqrt{\frac{2gh}{\left(\frac{1}{c}\right)^2 - \left(\frac{a}{A}\right)^2}}. \quad (33)$$

In this equation it will be noticed that V does not appear. Hence it is convenient to use in numerical problems where V is not known but the cross-section A is given. It should be noted that the velocity in the channel has been assumed as being uniform in all parts. This in reality is not so, but the error thus introduced is so slight as to be generally negligible.

Example. — Given,

$$h = 4 \text{ ft.}$$
$$a = 4 \text{ sq. in.}$$
$$A = 16 \text{ sq. ft.}$$
$$c = 0.60.$$

$$Q = a\sqrt{\frac{2gh}{\left(\frac{1}{c}\right)^2 - \left(\frac{a}{A}\right)^2}}$$

$$= \frac{4}{144}\sqrt{\frac{64.4 \times 4}{\frac{1}{0.36} - \left(\frac{0.0278}{16}\right)^2}},$$

$$Q = 0.27 \text{ c. f. p. s.}$$

Note. — Had Q been calculated *without* reference to the velocity of approach, the error would have been less than 1 per cent, and, in general, the error will be negligible if the area of the channel cross-section be at least 15 times the area of the orifice. This is easily proved by assuming values of a/A in our general formula and comparing the resulting Q's.

Example. — An orifice of $\frac{1}{2}$ sq. in. area is located in the end of the closed channel A-B (Fig. 53). A piezometer C-D contains water to a height of 4 ft. above the orifice. Assuming c as 0.60 and that the water in the pipe has a velocity of 1 ft. per second, find the discharge. Since the pressure at any point is measured by its depth below the free surface D, the pressure head at m is 4 ft., m being in the horizontal plane of the orifice. Writing Bernoulli's Theorem for points m and n,

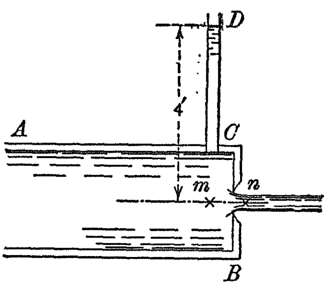


FIG. 53.

$$\frac{1}{64.4} + 4 + 0 = \frac{v^2}{64.4} + 0 + 0,$$

$$v = 16.1 \text{ f. p. s.}$$

$$Q = 0.60 \times \frac{.5}{144} \times 16.1 = 0.034 \text{ c. f. p. s.} \quad \textit{Ans.}$$

Problem. — Assuming that in the previous problem the open tube is replaced by an ordinary pressure gauge, find the discharge when the gauge registers 40 lb. per square inch.

46. Flow under Pressure. — If either reservoir surface or jet be under pressure other than atmospheric, the formula $Q = ca\sqrt{2gh}$ does not directly apply. However, as it is merely a change in pressure conditions at these points, the application of the general theorem will suffice for solving. For example, Fig. 54 shows in section a vessel partly filled with water and fitted with an air-tight piston. A force of 1000 lb. being applied, it is desired to find the velocity of flow from A at the moment when the piston is in the position shown. Assume $c_v = 0.98$ and the area of the piston = 1 sq. ft. For points m and n we have (neglecting velocity of approach),

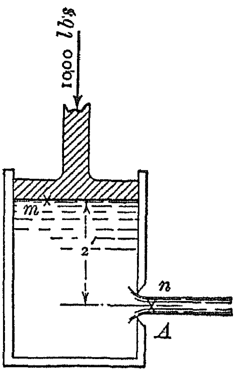


FIG. 54.

$$0 + \frac{1000}{62.5} + 2 = \frac{v^2}{64.4} + 0 + 0.$$

$$v = 34.3 \text{ f. p. s.}$$

$$\text{Actual } v = 0.98 \times 34.3 = 33.6 \text{ f. p. s.} \qquad \textit{Ans.}$$

Example. — What will be the velocity of flow from the orifice in the side of the tank (Fig. 55) when steam under pressure of 120 lb. per square inch fills the space above the water, and the receiving tank has in it a pressure of 4 lb. less than atmospheric? Again, Bernoulli's Theorem for points m and n gives

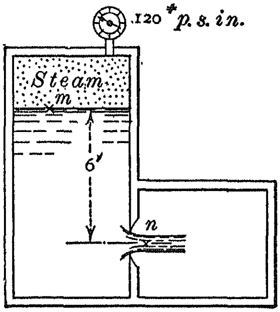


FIG. 55.

$$0 + \frac{134.7 \times 144}{62.5} + 6$$

$$= \frac{v^2}{64.4} + \frac{10.7 \times 144}{62.5} + 0.$$

$$v = 137 \text{ f. p. s.}$$

$$\text{Actual } v = 0.98 \times 137 = 134 \text{ f. p. s.}$$

47. Submerged Orifice. — If an orifice discharges wholly under water, it is said to be *Submerged*. That the theoretic velocity equals $\sqrt{2gh}$, where h is the difference in the water levels, can be easily shown, assuming atmospheric pressure only to be on the two surfaces. For the points m and n we have (Fig. 56)

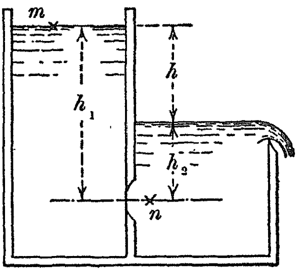


FIG. 56.

$$0 + 34 + h_1 = \frac{v^2}{2g} + (34 + h_2) + 0,$$

from which

$$v^2 = 2g\,(h_1 - h_2),$$

$$\text{or} \quad v = \sqrt{2gh}. \quad \text{Q.E.D.}$$

As before, $$Q = ca\sqrt{2gh},$$

but the values of c are now different from those given for the case of an orifice discharging into air. Experiment shows a slight decrease in the amount discharged, as may be noted in Table V, which is based on experiments made by Hamilton Smith in 1884.

Submerged orifices are common in engineering works, being found in locks, waste ways, tide gates, and many other constructions.

48. Standard Orifice. — Anything that tends to decrease the contraction beyond the orifice will cause an increase in the amount discharged per unit of time. So far we have been dealing with an orifice having sharp edges so that the water in passing touches only a line. The term *Standard* has been

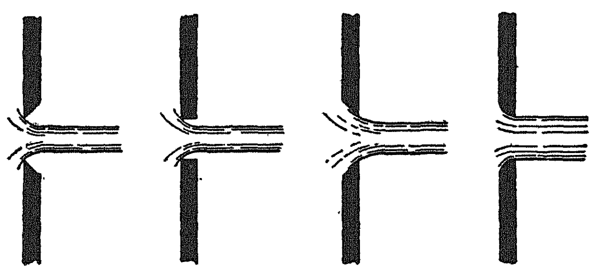


FIG. 57.

applied to such an orifice, as it is between orifices of this kind only that comparisons can easily be made. Figure 57 shows

various types of orifices of the same diameter. The first (*a*) is the above-mentioned standard orifice. The second (*b*) is a clean-cut hole in a plate of measurable thickness. It has the same coefficients as (*a*) provided its thickness is small. The third (*c*) is the reverse of (*a*) and, while it forms a contracted jet, its coefficient of discharge is much greater and depends on the shape of the bevels. The last (*d*) shows the inner edge carefully rounded to conform to the shape of the contracted vein. The probable coefficient of discharge is only a little less than unity. Since the coefficients vary from about 0.60 to nearly 1.0, depending on the shape of the orifice, it is very essential to use only standard orifices when endeavoring to accurately measure discharge.

49. Suppression of the Contraction. — The location of the orifice with respect to its distance from the sides and bottom of the reservoir is also a matter of importance. If it be flush with any one side, as in Fig. 58, the contraction on that side of the orifice will be wholly suppressed. Experiment has shown that the contraction is not fully restored until the orifice is moved far enough away from the side to provide a free lateral approach from all directions for a distance equal to three times the least dimension of the orifice. Orifices are sometimes improperly constructed, as in Fig. 59 *a* and the plate *A-B* prevents the lateral approach. A preferable construction would be that shown in Fig. 59 *b*. If the orifice were made in the side of a cylindrical tank, the curvature of the tank might slightly diminish the contraction.

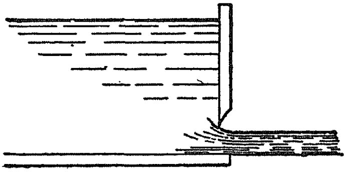


FIG. 58.

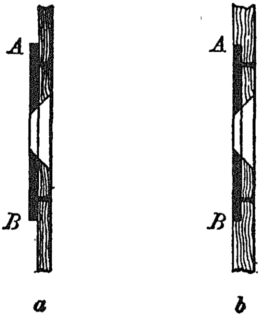


FIG. 59.

50. Water Measurement by Orifices. — Next to weighing, the orifice furnishes the most accurate way of measuring moderate volumes of water because of our knowledge of the value of

the coefficients. If all the precautions enumerated in the preceding articles be observed, the result should not be in error much over 1 per cent. Care is necessary in measuring the head, and for this purpose either a piezometer tube or a hook gauge may be used. If the head exceeds 2 feet, the piezometer will be found to furnish all the necessary accuracy, as the head may be easily read to one one-hundredth of a foot; and an error of this amount in reading the gauge at this head would result in an error in the figured discharge of less than one third of 1 per cent. As commonly used the tube is of glass connected with the tank at a point somewhat below the surface level by means of metal or rubber connections, and carried vertically upward to a point above the surface. On its sides, or attached to it, is a scale divided into feet, tenths, and hundredths of feet, with its zero at the orifice's center. The position of the lowest point of the curved meniscus forming the top of the water column is then easily read.

51. The Hook Gauge. — For low heads, or where great accuracy is required, use is made of the hook gauge in determining the height of the surface level. This may be briefly described as a brass rod arranged to move vertically between fixed supports and having at its lower end a sharp-pointed hook. The rod is controlled in its vertical movement by a slow-motion screw, and a vernier attachment enables its position to be read to the one-thousandth part of a foot. In using the gauge the hook is lowered beneath the surface of the still water and then raised until the point causes a tiny pimple or elevation to appear on the surface. When the hook is lowered sufficiently to cause this to just disappear, it is assumed to be in the level of the surface. The vernier is then read. The position of the hook, relative to the orifice's center, for zero reading of the vernier, must be carefully determined by previous measurement.

52. The Miner's Inch. — Mention must be made, in passing, of the manner of measuring water which has for years been in vogue in the western part of the United States. In the mining section water is commonly sold by the miner's inch, which may be approximately defined as the amount of water which will pass in

te : Energy equation applies only for Steady Flow.

24 hours a sharp-edged orifice, one inch square, under a head of $6\frac{1}{2}$ inches. Inasmuch as this is a relatively small quantity, it is customary to measure the water through orifices of much larger size and assume that as many inches are delivered as there are square inches in the orifice's area, the head being kept at 6 inches on the *upper edge*. This, with other practices connected with the mode of measurement, makes the miner's inch a variable and vague quantity; and it is to be hoped that gradually this unsatisfactory and unscientific method will be discarded and water sold by the cubic foot.

53. Discharge under a Falling Head. — It is often required to find the time necessary to empty a reservoir or draw down its level a certain amount. Let h_1 (Fig. 60) be the head on the orifice at the moment of opening and h_2 the head at the end of some interval of time t. We will assume the horizontal cross-section of the reservoir to be constant. At any particular instant the velocity of flow is

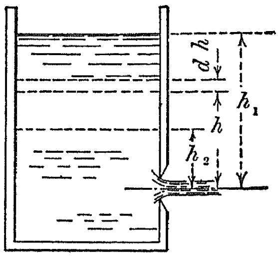


FIG. 60.

$$v = c_v\sqrt{2\,gh},$$

where h is the head at that instant. The quantity that would be discharged per second under that head is

$$Q = ca\sqrt{2\,gh},$$

and in a very small fraction of time dt a small quantity

$$dQ = ca\sqrt{2\,gh}\;dt$$

would be discharged. In the same small interval of time the head would drop a small amount dh, and the amount passing from the reservoir can be expressed as the drop times the cross-sectional area A, *i.e.*

$$dQ = dh \times A.$$

Equating these values of dQ,

$$-\,dhA = ca\sqrt{2\,gh}\;dt,$$

the minus sign being given to dh since it is a *decrement*, while dt is an *increment*.

$$dt = -\frac{A\,dh}{ca\sqrt{2\,gh}}.$$

Since h varies from h_2 to h_1, the integration of the above will give

$$t = -\frac{A}{ca\sqrt{2\,g}}\int_{h_1}^{h_2} h^{-\frac{1}{2}}\,dh$$

or

$$t = \frac{2\,A}{ca\sqrt{2\,g}}\left(h_1^{\frac{1}{2}} - h_2^{\frac{1}{2}}\right). \quad . \quad . \quad . \quad (34)$$

If the horizontal cross-section of the reservoir is not of constant area, the problem is still possible of solution if this variable area be expressed in terms of the other variable h.

For the prismatic reservoir we may determine the value of v_m, the mean velocity of flow under a falling head, as follows:

$$v_m = \frac{\text{Total } Q}{\text{Area of jet} \times \text{time}} = \frac{A\,(h_1 - h_2)}{c_c a \times \dfrac{2\,A\,(\sqrt{h_1} - \sqrt{h_2})}{c_d a\sqrt{2\,g}}},$$

or

$$v_m = c_v\sqrt{2g}\,\frac{(h_1^{\frac{1}{2}} + h_2^{\frac{1}{2}})}{2} = \frac{c_v\sqrt{2\,gh_1} + c_v\sqrt{2\,gh_2}}{2}.$$

Thus we find that the *mean* velocity is the arithmetical mean of the velocity due to h_1 and that due to h_2. This simple relation will make the solution of many problems under falling head very easy.

Example (From Bovey). — One of the locks on the Lachine Canal has a superficial area of 12,150 sq. ft., and the difference of level between the surfaces of the water in the lock and in the upper reach is 9 ft. The gate between is supplied with two sluices and the water is leveled up in 2 min. and 48 sec. Determine the proper area of each sluice opening, assuming a discharge coefficient of 0.60. (The sluices are situated 20 ft. below the upper level.)

By formula (34),

$$168 = \frac{24300}{0.6 \times a \times 8.02}(\sqrt{9} - \sqrt{0}),$$

or $$a = 90 \text{ sq. ft.}$$

As this is the combined area of the two sluices, each must have an area of 45 sq. ft. *Ans.*

Example. — A prismatic vessel (Fig. 61) has two compartments A and B, communicating by a standard orifice 6 inches square, its center being 3 feet above the bottom of the vessel. The horizontal cross-section of A is 100 sq. ft., and that of B is 200 sq. ft. At a certain time the water stands 13 ft. deep in A and 9 ft. deep in B. How soon thereafter will the surfaces reach a common level?

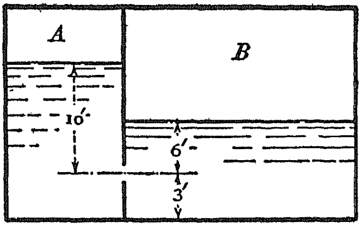


FIG. 61.

Let y be the depth of water in A at any instant, and dy be the change in depth during any interval of time dt. The rise in B's level during the same time will be $dy\ (100 \div 200)$, and the net change, dh, in head will be

$$dh = \tfrac{3}{2}\, dy. \qquad (dy + \tfrac{1}{2} dy)$$

Through the orifice there flows in dt seconds

$$dQ = ca\sqrt{2gh}\ dt;$$

also $$dQ = 100\, dy = \tfrac{200}{3}\, dh.$$

$$\therefore\ -\tfrac{200}{3}\, dh = ca\sqrt{2gh}\ dt = 0.6 \times \tfrac{36}{144} \times 8.02 \times h^{\frac{1}{2}}\, dt.$$

$$t = -55.4\int_4^0 h^{-\frac{1}{2}} dh.$$

Note the limits of integration?

$$t = 221.6 \text{ sec.} \quad \textit{Ans.}$$

54. Head lost in an Orifice. — To ascertain how much head is lost in friction as water passes an orifice, we may proceed as follows.

Theoretically	*Actually*
$v = \sqrt{2\,gh}.$	$v_a = c_v\sqrt{2\,gh}.$
$\dfrac{v^2}{2\,g} = h.$	$\dfrac{v_a^{\,2}}{2\,g} = c_v^{\,2}\,h.$

The value of the lost head must therefore be

$$\text{Lost head} = h - c_v^{\,2} h = (1 - c_v^{\,2})\,h. \quad . \quad . \quad (35)$$

For the standard orifice, ($c_v = 0.98$); this would give

$$\text{Lost head} = 0.040\,h.$$

If for h in equation (35) we substitute its value in terms of the *actual* velocity, there results

$$\text{Lost head} = \left(\frac{1}{c_v^{\,2}} - 1\right)\frac{v_a^{\,2}}{2\,g}. \quad . \quad . \quad . \quad (36)$$ Impt.

For the standard orifice,

$$\text{Lost head} = 0.041\,\frac{v_a^{\,2}}{2\,g}.$$

Note. — Equations (35) and (36) are applicable to any discharging device whose velocity coefficient is known.

PROBLEMS

1. A steel box, rectangular in plan, floats with a draft of 2 ft. If the box be 20 ft. long, 10 ft. wide, and 6 ft. deep, find the time necessary to sink it by opening a standard orifice, 6 in. square, in its bottom. (Neglect thickness of sides.)

2. Water spurts out horizontally from a half-inch round hole in the side of a timber penstock, the wall of which is 2 in. thick. The jet strikes at a point 6 ft. horizontally distant from the orifice and 2 ft. lower down. Find the probable leakage in cubic feet per 24 hours.

3. The jet from a circular, sharp-edged orifice $\frac{1}{2}$ in. in diameter, under a head of 18 ft., strikes a point distant 5 ft. horizontally and 4.665 in. vertically from the orifice. The discharge is 98.987 gal. in 569.218 seconds. Find the coefficients of discharge, velocity, and contraction. (Bovey's "Hydraulics.")

4. The piston of a vertical 12-in. cylinder containing water is pressed down with a total effective force equal to 3000 lb. What velocity of flow should theoretically be obtained from an orifice in the cylinder 20 ft. below the piston?

5. Find the velocity of discharge through an orifice in the bottom of a vessel moving upward with an acceleration of 10 ft. per second per second, the depth of water being 8 ft.

6. A circular orifice 8 in. in diameter is made in the side of a reservoir into which water flows at the rate of 6 cu. ft. per second. Determine the height above the center of the orifice to which the water will rise in the reservoir.

7. A vessel 16 ft. high and 6 ft. in diameter contains water 7 ft. deep when at rest. How fast must it be revolved about an axis through its center in order that the relative velocity of flow from an orifice in the side, and 1 ft. above the bottom, may be 24 ft. per second?

8. The head in a prismatic vessel at the instant of opening an orifice was 9 ft. and at closing had decreased to 5 ft. Determine the constant head under which, in the same time, the orifice would discharge the same volume of water.

9. A reservoir half an acre in area, with sides nearly vertical, so that it may be considered prismatic, receives a stream yielding 9 cu. ft. per second, and discharges through a sluice 4 ft. wide, which is raised 2 ft. Calculate the time required to lower the surface 5 ft., the head over the center of the sluice, when opened, being 10 ft. (Bovey's "Hydraulics.")

10. A reservoir of water is connected with one containing oil by a sharp-edged circular orifice 2 in. in diameter. Determine the rate of discharge when the water stands 6 ft. and the oil 2 ft. deep on the orifice's center. The oil has a specific gravity of 0.8.

11. Water spurts vertically upward from a horizontal orifice under a head of 40 ft. What is the limit to the height of the jet if the coefficient of velocity be 0.97?

12. A fluid of one fourth the density of water is discharged into the free atmosphere from an orifice in the side of a reservoir. If a pressure of 50 lb. per square inch (absolute) exists just back of the orifice, find the theoretic velocity of discharge.

13. Find the theoretic discharge through a vertical, sharp-edged orifice having the form of an isosceles triangle with an altitude of 10 in., base (horizontal) 6 in., and vertex touching the water level in the reservoir. Disregard velocity of approach.

14. Water completely fills a vessel whose shape is that of an inverted pyramid with a square base. If the base measures 12 in. on a side and the altitude is 2 ft., how long a time will be required to empty the vessel through an orifice, 1 in. in diameter, at the apex? Assume $c = 0.60$.

CHAPTER VI

FLOW THROUGH MOUTHPIECES

55. General. — If in any manner a mouthpiece be added to an orifice, we may expect changes in both velocity and quantity discharged. As for velocity, we notice a general diminution on account of increased frictional resistances, eddyings, impact, etc. The quantity discharged is either increased or diminished, according to the construction of the mouthpiece.

56. Standard Short Tube. — If a short cylindrical tube, having a length of 2 to $2\frac{1}{2}$ diameters, be attached to the orifice on its outer side, we have what is known as the standard short tube (Fig. 62). If the flow be started by first stopping the

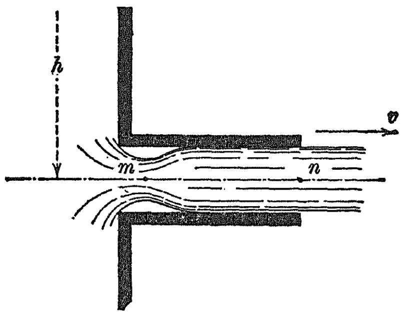


FIG. 62.

tube at exit, and then allowing the water to escape, the jet will issue in parallel filaments, suffering no contraction, and completely filling the tube at the end. Inside the tube at m the jet is contracted by passing the sharp edges of the opening, but quickly expands to fill the tube. By reason of the eddying, which follows the expansion, considerable friction is developed. This, added to friction at entrance and along the sides of the tube, tends to materially reduce the velocity. If Bernoulli's

Theorem be applied between a point in the reservoir surface and one in the free jet, we obtain, as we did for the orifice,

$$v = \sqrt{2gh}.$$

Weisbach obtained from experiment a mean value of

$$v = 0.815 \sqrt{2gh}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

the coefficient varying slightly with the head and diameter of tube. We may then write

$$Q = 0.82\, a \sqrt{2gh},$$

inasmuch as the contraction coefficient is unity. Thus we have a discharge which is one third greater than that obtained from the standard orifice, although the *velocity* of the discharge is much less.

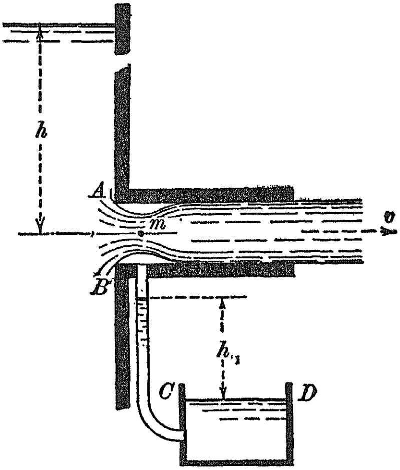


FIG. 63.

If a tube be tapped into the mouthpiece at the point where the contraction takes place, and its lower end be immersed in water (Fig. 63), the latter will rise in the tube, showing that the pressure in the contracted section is less than atmospheric. It will be interesting to determine the relation between the height of the water column h_1 and the static head h on the mouthpiece.

Between a point in the reservoir surface and one at m we may write

$$0 + \frac{p_a}{w} + h = \frac{v_m^2}{2g} + \frac{p_m}{w} + 0 + \text{Lost head.}$$

and if v_m and the lost head can be expressed in terms of h, this general equation may be solved for $\frac{p_m}{w}$ in terms of h. The

value of $\frac{v_m^2}{2g}$ may be found by assuming a coefficient of contraction for the stream at m. In the case of a standard orifice this is about 0.62; but there is reason to believe that here the contraction is slightly less, due to less pressure on the section, and possible backing up of the water beyond. We will assume $c_c = 0.64$. Then, with steady flow,

$$v_m \times 0.64\, a = v \times a,$$

or $$v_m = 1.56\, v = 1.56 \times 0.82\sqrt{2\, gh} = 1.28\sqrt{2\, gh},$$

from which $$\frac{v_m^2}{2\, g} = 1.64\, h.$$

We have next to determine the value of the head lost in friction as the water enters the tube past the edge *A-B*, this being the only sensible loss occurring between the points chosen. This is best done by considering for a moment the case of the standard orifice and the similar loss occurring there. This we saw, in Art. 54, equation (36), was

$$\text{Lost head} = 0.041 \frac{v^2}{2\, g},$$

the loss being expressed in terms of the velocity head found in the issuing jet.

Returning to the short tube, it seems reasonable to believe that the loss at the edge *A-B* will be closely $0.041 \times$ the velocity head in the *contracted section*, or

$$\text{Lost head} = 0.041 \times 1.64\, h = 0.07\, h. \quad . \quad . \quad (38)$$

Substituting in the general equation the values now found for $\frac{v_m^2}{2\, g}$ and the lost head, and remembering that $\frac{p_a}{w} = 34$ ft., we obtain

$$0 + 34 + h = 1.64\, h + \frac{p_m}{w} + 0.07\, h,$$

or $$\frac{p_m}{w} = 34 - 0.71\, h. \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

That is to say, the pressure head at the contracted section is less than atmospheric by an amount equal to $0.71\, h$. This latter

is the height of the water in the tube, and we have now the relation

$$h_1 = \tfrac{3}{4} h \text{ (approx.)}.$$

The correctness of the relation and our assumptions was demonstrated by Venturi. With a head of 0.88 meters he obtained a water column of 0.65 meters, giving a relation of $h_1 = 0.74h$.

Referring to Fig. 63, it will be seen that if the distance B-C is less than h_1, the water will enter from the glass tube into the mouthpiece and be expelled with the jet. This is the principle of the jet pump.

Furthermore, as the height of the water column cannot exceed 34 ft. (approx.), it follows that the pressure in the contracted section will be absolute zero when the static head is about 45 ft. (since $34 = \frac{3}{4} h$). Experiments conducted in the Hydraulic Laboratory of the Massachusetts Institute of Technology in 1900 showed that when the head approached or exceeded about 42 ft. the flow became "troubled and pulsatory." This was probably due to the breaking down of the contracted section when all pressure was removed.

At Cornell University some experiments were made on tubes which had been made of such a form internally as to conform to the shape of the contraction (Fig. 64). It was found that the flow was increased about 10 per cent in these tubes, which may be explained by saying that the eddying would be diminished and some loss by friction prevented.

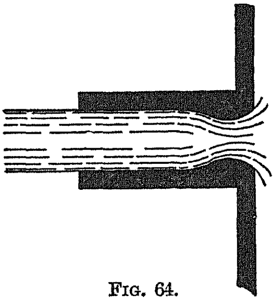


FIG. 64.

The short tube is of practical importance only as it serves later to aid in the study of flow through long pipes. As a device for measuring water, it is much less reliable than the orifice, since little is known about its coefficient, as the diameter of the tube and the head vary.

57. Conical Converging Tube. — This mouthpiece is of special interest, as it approximates the nozzle used for fire and engineer-

ing purposes. It consists of a frustum of a hollow cone with the larger end placed at the orifice (Fig. 65). From such a tube the water flows in a jet having a slightly contracted section a-a just beyond the tip. The amount of contraction being very slight, the contraction coefficient is not far from unity and will increase as the angle of convergence β decreases. The velocity coefficient, on the other hand, is found to diminish as β decreases, and has its maximum value of 0.98 when $\beta = 180°$. The tube under this condition becomes an orifice. These variations are well shown in the accompanying table, based on the work of D'Aubisson and Castel, who experimented with this type of mouthpiece.

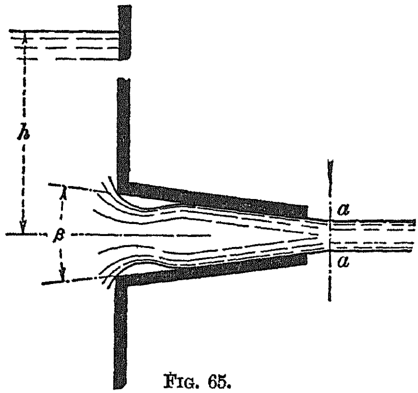


FIG. 65.

ANGLE β	0° 0′	3° 10′	7° 52′	10° 20′	13° 24′	19° 28′	29° 58′	48° 50′
c_v	0.829	0.894	0.931	0.950	0.962	0.970	0.975	0.984
c_c	1.000	1.001	0.998	0.987	0.983	0.953	0.919	0.861
c	0.829	0.895	0.929	0.938	0.946	0.924	0.896	0.847

The coefficients of discharge were obtained by direct measurement of the volumes of water discharged, and the velocity coefficients were calculated by measuring the path of the jet. (See Art. 42.) The contraction coefficients were then deduced from $c_c = c \div c_v$. The method of obtaining the coefficient of velocity makes no allowance for the retardation of the particles in the jet by the atmosphere; hence the coefficients may be a little small, and the contraction coefficients would then be too large as deduced from the above equation. This would seem to account for the value of 1.001, as seen in the second column. The table would indicate that the greatest discharge occurred when the angle of convergence was near 13°, and, as

the corresponding velocity was relatively high, this angle should give a very efficient nozzle.

58. Nozzles for Practical Purposes. — A nozzle, as used for fire or engineering purposes, is a modification of the convergent tube, shaped to do away with the contraction in the free jet. This is generally accomplished by the addition of a cylindrical tip, as shown in Fig. 66 *a*. This does not materially alter the velocity of the jet, but does increase the discharge. The tip is given a length sufficient to prevent the water passing through without wetting it. Sometimes the whole nozzle is made convex on the inside, so as to give to the particles a parallel motion as they leave the tip (Fig. 66 *b*). These two types of nozzles are most commonly used in practice, as it is found that they furnish the most efficient streams. The experiments of Freeman indicate that for either kind the mean value of the discharge coefficient is very near to 0.97. For fire streams they should be particularly adapted, as they would throw large quantities of water through a maximum of distance. Among other types which have been proposed and used to a limited extent is the so-called "ring nozzle." This is made with an annular ring inside the tip (Fig. 67) which causes the stream to contract as it leaves the nozzle. No good reason can be offered for its adoption, as the coefficient of discharge is made low (about 0.74) by the presence of the contraction.

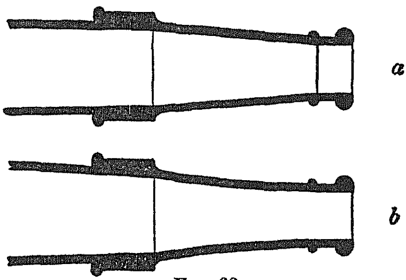


FIG. 66.

FIG. 67.

For all classes of nozzles we may write for the actual velocity of the jet,

$$v = c\sqrt{2\,gh},$$

and for the quantity discharged per second,

$$Q = ca\sqrt{2\,gh},$$

the values of c_v and c for any particular nozzle being determined experimentally.

Nozzles are generally used on the ends of pipe lines in which there may exist a considerable velocity of approach. In this case we may proceed as in the case of the orifice (Art. 45) and obtain,

$$v = c_v\sqrt{\frac{2\,gh}{1-\left(\frac{ac}{A}\right)^2}}.$$

The quantity discharged is

$$Q = a\sqrt{\frac{2\,gh}{\left(\frac{1}{c}\right)^2-\left(\frac{a}{A}\right)^2}}. \quad . \quad . \quad . \quad \text{(Art. 45)}$$

Here a would represent the area of the nozzle tip and A the area of the pipe. Since

$$\frac{a}{A} = \frac{d^2}{D^2},$$

d and D being the respective diameters, we may write

$$v = c_v\sqrt{\frac{2\,gh}{1-c^2\left(\frac{d}{D}\right)^4}}, \quad . \quad . \quad . \quad . \quad . \quad (40)$$

and

$$Q = a\sqrt{\frac{2\,gh}{\left(\frac{1}{c}\right)^2-\left(\frac{d}{D}\right)^4}}. \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Of course if the velocity of approach be known in value, the direct use of Bernoulli's Theorem between a point in the pipe and one in the jet will give a theoretical value for v which may be corrected by the coefficient.

As a device for measuring water, the nozzle takes rank next to the standard orifice and has the advantage of a greater capacity. The student is referred to the experiments of Freeman, reference to which is given at the end of this chapter, for details regarding the proper coefficient for use.

59. Diverging Tubes. — It has been pointed out that the chief loss of head in the standard short tube is occasioned by the sudden enlargement of the contracted vein to fill the full section of the tube. If the interior of such a tube should be shaped as in Fig. 68, so as not only to fit the form of the contracted vein but also to cause a *gradual* enlargement, this loss should be largely done away with and the discharge increased. Venturi and Eytelwein used such tubes and also rounded the edge at the point of entrance. Their tube was 8 inches long, 1 inch in diameter at the smallest section, and 1.8 inches in diameter at the large end. The angle of flare was 5° 9′ (Fig. 69). They obtained in their experiments a discharge nearly $2\frac{1}{2}$ times as great as though it had occurred through a standard orifice of the same diameter as the small section, and almost twice what would have been discharged by a standard short tube of that diameter. The velocity past m was much greater than that due to the head it was under, and the pressure at m was consequently less than one atmosphere. This last may be proved by writing Bernoulli's Theorem between m and n. The two points being on the same level and the velocity head at m greater than at n, the pressure head at m must be smaller than at n. The increased discharge through the small section is therefore due to the diminished pressure upon it, and if by any means atmospheric pressure were maintained there, the phenomenon of increased flow would be lost. Venturi proved this by boring holes through the sides of the tube at this point.

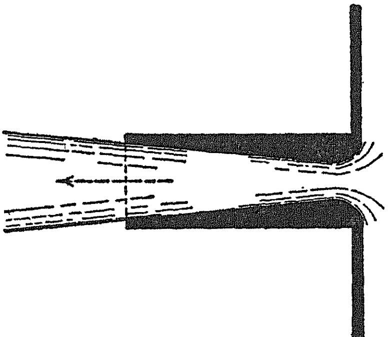


FIG. 68.

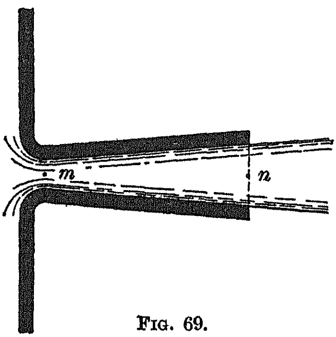


FIG. 69.

If the flare of the tube be considerable or the head upon it low, some trouble may be had to keep it flowing full at the outer end. To remedy this it is sometimes made to discharge under water as in Fig. 70. If the head on the outer end be h_2, then the effective head must be $h = h_1 - h_2$ just as for the submerged orifice.

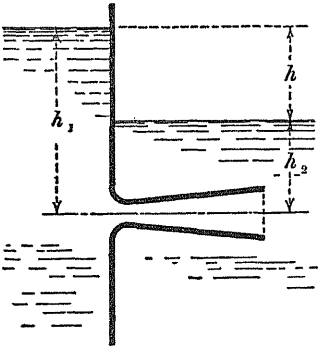


FIG. 70.

The principle of the flaring tube finds its application in the "diffuser," which is used with some types of turbine wheels to reduce the pressure at the outlet of the wheel. (See Church's "Motors," p. 126.) This tube commonly has the name of Venturi given to it in recognition of that distinguished experimenter.

59 *a*. Borda's Mouthpiece. — Mention is made here of this mouthpiece, not because of its practical importance as a measuring device, but because it presents the only case allowing a theoretical determination of the contraction coefficient. As shown in Fig. 70 *a*, it consists of an orifice fitted with a short re-entrant tube having a length about equal to its diameter. The issuing jet touches only the inner edge. To determine the amount of the contraction we may proceed as follows.

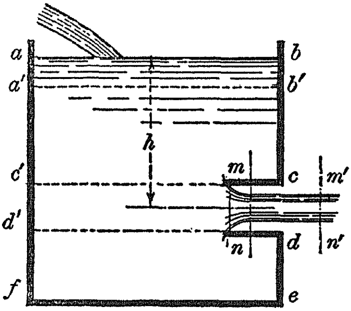


FIG. 70 *a*.

On the portions *bc* and *de* of the reservoir walls, the pressure at any point is practically the same as it would be under static conditions, since the presence of the tube prevents the particles in close proximity to the walls from having any appreciable velocity. The total pressure on *af* is therefore in excess of that on *be* by the amount, *awh*, that would exist on *c'd'*, the projected area of the orifice. Considering now the mass of

water between ab and mn, we have this same unbalanced pressure exerted upon it by the wall af. If in a time dt the mass moves to $a'b'm'n'$ and in so doing has its momentum in a horizontal direction changed, we may equate this change to the product of the horizontal force, awh, which produced it, and the time in which it was produced. That is,

$$awhdt = Mdv.$$

The change in momentum may be expressed as the difference in horizontal momentum of the small masses $aba'b'$ and $mnm'n'$. That this is so may be seen if it is noted that the aggregate momentum of all particles between $a'b'$ and mn remains constant. (A more detailed proof might be given similar to the discussion on page 44 relative to the change in kinetic energy taking place in the mass A–B, Fig. 39). For $aba'b'$ the horizontal momentum is zero, and that of $mnm'n'$ is

$$\text{Mass} \times \text{velocity} = \frac{w}{g}\,(c_c a)\,vdt \times v.$$

We may therefore write

$$(awh)\,dt = \frac{w}{g}\,c_c av^2 dt,$$

from which $$c_c = h \div \frac{v^2}{g} = \frac{v^2}{2\,g} \div \frac{v^2}{g} = 0.5.$$

This value is a theoretical one, since in substituting $\frac{v^2}{2\,g}$ for h no allowance was made for loss in velocity due to friction. If c_v be assumed as 0.98, we obtain

$$c_c = 0.52.$$

A condition of flow quite similar to that into a re-entrant tube occurs when a pipe is inserted through a reservoir wall and made to project inwardly.

PROBLEMS

1. A horizontal diverging tube such as is shown in Fig. 70 has a diameter of 3 in. at the contracted section and 4 in. at its mouth. Water stands 16 ft. deep above its axis on the reservoir side, and 2 ft. deep on its discharging end. Determine the pressure existing in the contracted section during flow, assuming that 2 ft. of head is lost in the tube between the point of entrance and point of discharge.

2. If in the previous problem a vertical tube, 1 in. in diameter, were tapped into the contracted section and made to communicate with a second reservoir, whose level was 8 ft. below the axis of the tube, at what velocity would water flow from this tube into the larger one?

3. If a 3-in. pipe, in which the *total head* is 40 ft., terminates in a flaring mouthpiece, what will be the maximum diameter possible at its outer end, if it is to discharge full bore into free air? Assume no loss by friction.

4. A 2-in. nozzle is attached to a 6-in. pipe line. Pressure at the base of the nozzle during flow is 50 lb. per square inch. The coefficient of discharge of the nozzle is 0.90. With these data compute the horse power of the jet.

5. A $1\frac{1}{4}$-in. nozzle, attached to a horizontal $2\frac{1}{2}$-in. play pipe, discharges 290 gallons a minute under an indicated pressure of 40 pounds per square inch at base of play pipe. What is the coefficient of discharge of the nozzle?

6. A nozzle points vertically downward and terminates in a $1\frac{3}{8}$-in. diameter orifice. It is supplied through a $2\frac{1}{2}$-in pipe, to which is attached a gauge 3 ft. above the nozzle orifice. When the gauge registers 30 pounds, the discharge is found to be 310 gallons per minute. Find the head lost between the gauge and the orifice.

CHAPTER VII

FLOW OVER WEIRS

60. Definitions.—In general terms a weir is a notch cut in the upper edge of a vertical wall, through which water is allowed to flow for purposes of measurement. The shape of the notch may vary, but the type of the weir which is most common has a rectangular notch with one edge horizontal. If the bottom and sides of the rectangle be far enough removed from the other walls of the reservoir to permit free lateral approach of the water in the plane of the weir, the stream issues from the notch contracted on these three sides and we have what is known as the Contracted Weir (Fig. 71). In contradistinction to this type is the Suppressed Weir (Fig. 72), in which the end contractions due to the vertical edges are suppressed by making the ends of the notch coincident with the side walls of the reservoir. The contraction is said to be *complete* and *incomplete* in the respective cases. In both weirs the surface of the water immediately back of the weir does not stand level, but assumes a curve as indicated in Fig. 73. The horizontal edge of the weir is known as the *Crest;* and the *Head* on the weir is not the depth of the water vertically over the crest, but is measured as shown from the level of the crest up to the general reservoir surface.

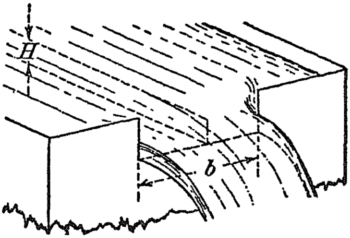


FIG. 71.

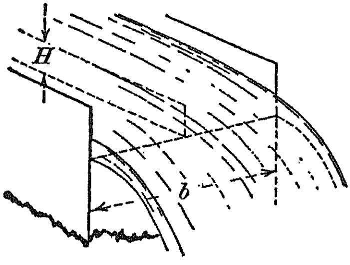


FIG. 72.

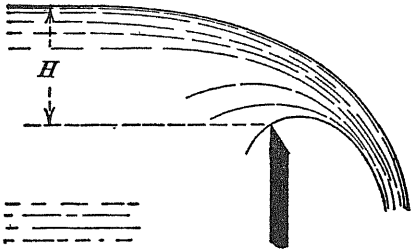


FIG. 73.

61. Fundamental Equations for Flow over Rectangular Weirs. — If a large vertical orifice discharges with a head on its upper edge equal to zero, we have results analogous to the flow over a weir. Let Fig. 74 represent in section a large rectangular orifice in the end wall of a reservoir or channel, the latter being so shaped or proportioned as to produce a velocity

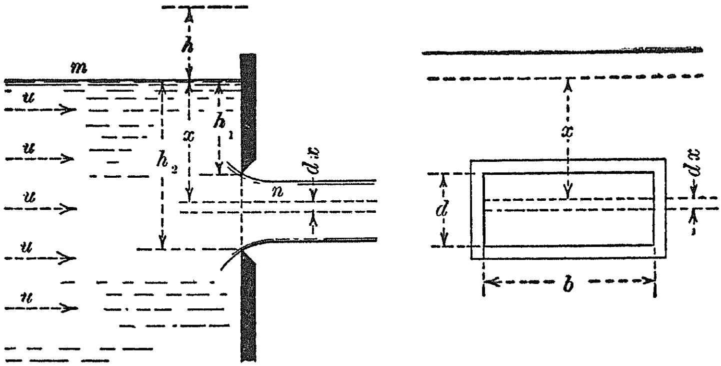


FIG. 74.

of approach u. If v represents the velocity of discharge through any horizontal elementary strip situated a distance x below the surface, the discharge per unit time is

$$dQ = dA \cdot v = b \cdot dx \cdot v,$$

and for the entire orifice,

$$Q = b\int v \cdot dx.$$

To express v in terms of x, Bernoulli's Theorem may be written between the points m and n, giving

$$\frac{u^2}{2g}+\frac{p_a}{w}+x=\frac{v^2}{2g}+\frac{p_a}{w}+0,$$

and
$$v=\sqrt{2g}\,(x+h)^{\frac{1}{2}},$$

where h has replaced $\frac{u^2}{2g}$. Substituting this value for v in the above expression for Q, there results

$$Q=b\sqrt{2g}\int(x+h)^{\frac{1}{2}}dx.$$

The quantity $(x+h)$ may be regarded as a variable with limits of (h_1+h) and (h_2+h) inasmuch as $d(x+h)=dx$. With these limits the above equation becomes, when integrated,

$$Q=\tfrac{2}{3}b\sqrt{2g}\,[(h_2+h)^{\frac{3}{2}}-(h_1+h)^{\frac{3}{2}}].$$

By making h_1 equal to zero, the orifice becomes a weir, and, replacing h_2 by H, we obtain

$$Q=\tfrac{2}{3}b\sqrt{2g}[(H+h)^{\frac{3}{2}}-h^{\frac{3}{2}}]. \quad . \quad . \quad . \quad (42)$$

If there be no velocity of approach, $h=0$ and

$$Q=\tfrac{2}{3}b\sqrt{2g}H^{\frac{3}{2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

These two equations will give the *theoretical* discharge for either a contracted or suppressed weir and may be regarded as fundamental equations. To apply either equation in practice it is necessary that a coefficient be introduced to care for frictional disturbances and contractions at the weir edges. To ascertain the value of this coefficient under different conditions of head, length, and type of weir has been the aim of many experimenters. They have tried also to deduce other formulæ with an aim to simplicity and wide range of application. No objection can be raised to such formulæ *provided* they have a rational basis and are applied only when the conditions under which they are used fall within the limits of the observations by which they and their coefficients were obtained. The best of these will now be discussed.

62. Francis' Formulæ. — In 1851, at Lowell, Mass., Mr. James B. Francis carried on an extensive series of experiments with large rectangular weirs. Most of the weirs were approximately 10 feet long and the heads ranged from 0.6 to 1.6 feet. As the result of many experiments he proposed the two following equations,

$$Q = 0.622 \times \frac{2}{3}\left(b - \frac{nH}{10}\right)\sqrt{2g}\left[(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right], \quad (44)$$

and

$$Q = 0.622 \times \frac{2}{3}\left(b - \frac{nH}{10}\right)\sqrt{2g}\, H^{\frac{3}{2}}, \quad . \quad . \quad (45)$$

as being adapted to rectangular weirs, with or without end contractions. The first (44) provides for the effect of the velocity of approach, and the second is to be used when this is so small as to be negligible. They may be regarded as identical with our fundamental formulæ (42) and (43), save that Francis assumed the *effective* length of the weir to be less than b by an amount $\frac{nH}{10}$, due to end contractions. The quantity 0.622 is the value of the coefficient c, and n varies in value as follows: —

$n = 2$ when there are two complete end contractions.
$n = 1$ when one end contraction is entirely suppressed.
$n = 0$ when both end contractions are suppressed.

The equations may therefore be written: —

(*a*) *Contracted Weir.* —

$$Q = 3.33\left(b - \frac{2H}{10}\right)\left[(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right], \text{with vel. of approach. } (46)$$

$$Q = 3.33\left(b - \frac{2H}{10}\right)H^{\frac{3}{2}}, \text{ no vel. of approach. } . \quad (47)$$ ✓ Im

(*b*) *Suppressed Weir.* —

$$Q = 3.33\, b\left[(H+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right], \text{ with vel. of approach. } . \quad . \quad (48)$$ ✓ Im

$$Q = 3.33\, bH^{\frac{3}{2}}, \text{ with no vel. of approach. } . \quad . \quad (49)$$ ✓ Imp

Francis' formula if p is ≥ 3H, neglect velocity of

Francis' formulæ have been very widely used in this country, where for years they were the only ones based on extensive and reliable experiments. They are simple of application, since they require no set of tabulated coefficients for varying conditions of head and length of weir. From over eighty experiments he deduced a mean value for c of 0.622, which varied from the extremes by about 3 per cent. This latter figure would therefore indicate the probable limit of the error resulting from applying his formulæ to weirs that closely approximate, in length and head, those upon which he experimented. His studies led him to conclude that the proportional amount of contraction on the top and bottom side of the issuing stream was constant at all heads, while the amount of the end contractions (when present) varied with the head. He accordingly made his coefficient c to include the effect of friction and *vertical* contraction only, thus obtaining a fairly constant value for it. To care for the end contraction he proceeded as follows: Since this appeared to vary with the head, it seemed logical to reason that the amount by which one end contraction diminished the discharge would be *constant* for any head. Its presence, therefore, would have the result of decreasing the effective length of the weir, and the quantity $\left(b - \frac{nH}{10}\right)$ represents the value of the effective length as deduced from his experiments. The value of n has already been given.

Francis found that the curvature of the end contraction extended horizontally over a distance of $1.5\,H$, and consequently recommended that, for a weir with two end contractions, the length b be greater than $3\,H$. Otherwise the effect in the discharge of side and bottom contractions would not be properly represented by his coefficient and the quantity $\left(b - \frac{nH}{10}\right)$.

A direct solution of equation (46) or (48) necessitates a knowledge of h, whose value we have seen to be $\frac{u^2}{2g}$. Since u, the velocity of approach, cannot in general be measured directly and the corresponding value for h determined, we are obliged to use a method of trial in computing the discharge. The

approximate discharge may be first calculated by neglecting the velocity of approach and using (47) or (49). The velocity in the channel may then be approximated from

$$u = \frac{Q}{A},$$

A = area of channel — [illegible]. See Ex. below.

A being the area of the channel's cross-section and u the average velocity past the section. If h be determined from this value of u, equation (46) or (48) may be used to obtain a new value for Q. This will be nearer to the true Q, but may still be somewhat approximate if the velocity of approach be considerable. By using this value to again find u and h, formula (46) or (48) may be used the second time and a value of Q obtained which will differ so little from the previous one that a recalculation of h would fail to materially alter the discharge. This method of correcting for the velocity of approach should be strictly adhered to in using Francis' formulæ, since it was the method used by him when determining the value of his coefficient.

Example. — A suppressed weir having a crest 10.58 feet long discharges under a head of 0.682 feet. If the depth of the channel of approach be 2.2 feet, find the discharge per second. Neglecting the velocity of approach,

$$Q = 3.33 \times 10.58 \times (0.682)^{\frac{3}{2}}$$

$$Q = 19.84 \text{ cu. ft. per second.}$$

$$u = \frac{Q}{A} = \frac{19.84}{10.58 \times 2.2} = 0.85 \text{ ft. per second.}$$

$$h = \frac{(0.85)^2}{64.4} = 0.011 \text{ ft.}$$

Using formula (48),

$$Q = 3.33 \times 10.58 \left[(0.693)^{\frac{3}{2}} - (0.011)^{\frac{3}{2}} \right]$$

$$Q = 20.3 \text{ cu. ft. per second.}$$

This may be taken as the final value, inasmuch as a recalculation of h gives 0.012, and this differs so slightly from the value used as to make resubstitution unnecessary.

63. Fteley and Stearns' Formulæ. — These two engineers experimented at Framingham, Mass., in 1877 and 1880, with results differing from those of Francis. Their experiments indicated that the end contraction did not depend wholly on the head, but was quite irregular, varying also with the velocity of approach. For this reason they recommended the use of weirs with end contractions suppressed, stating it to be their belief that end contractions were a source of error. For suppressed weirs they proposed

$$Q = 3.31\,bH^{\frac{3}{2}} + 0.007\,b, \tag{50}$$

to be used when the velocity of approach is negligible, and

$$Q = 3.31\,b\,(H + 1.5\,h)^{\frac{3}{2}} + 0.007\,b, \quad . \quad . \tag{51}$$

when it is so large as to influence the discharge.

The correction for the velocity of approach is made in (51) by adding $1.5\,h$ to the measured head, h being derived as for Francis' formula from $h = \frac{u^2}{2\,g}$. This method of correction was based on the fact that the water as it approaches the weir has not the same velocity in all parts of the channel's cross-section as would be obtained from $u = \frac{Q}{A}$. The stream lines near the surface and center of the channel are found to have a higher velocity, depending on the width and depth of stream and also the *height of the weir crest* above the channel. It is the velocity of this water, especially in the case of a contracted weir, that largely influences the discharge; and there would seem to exist good reason for increasing the value of h as found from the *mean* velocity. While Fteley and Stearns adopted the value of $1.5\,h$, they stated it was to be regarded as a mean value only, and furnished a table from which closer values might be selected. This is reproduced in part in Table VI of the Appendix and should be studied by the student with reference to the change in coefficient as the head, and also the depth of water below the crest, increases. From their experiments on contracted weirs they believed that these same formulæ could be applied pro-

vided b was replaced by $\left(b - \frac{nH}{10}\right)$ as in the Francis formula. To allow for the velocity of approach the effective head should be made equal to $(H + 2.05\,h)$, the numerical coefficient 2.05 being again a mean value. As stated previously, however, they regarded end contraction as a source of error and recommended its avoidance.

The Fteley and Stearns formulæ are based upon experiments on weirs from 5 to 20 feet in length, discharging under heads ranging from 0.07 to 1.63 feet. They should be used preferably on weirs falling within these limits, although their authors stated they believed the head could be increased "to as large depths as could be properly corrected for the effect of velocity of approach." With no appreciable velocity of approach these formulæ give results differing but little from those of Francis. With small velocities the results are nearly identical, while with large velocities Francis' formula gives smaller results.

64. Bazin's Formula for Suppressed Weirs.—In 1888 H. Bazin of France began a series of experiments on weir flow that have since received wide recognition on account of the masterly way in which he carried them out. His experiments and studies led him to propose for a suppressed weir, having no velocity of approach,

$$Q = cb\sqrt{2g}\,H^{\frac{3}{2}}, \qquad (52)$$

and for c he deduced the value,

$$c = 0.405 + \frac{0.00984}{H},$$

stating that for heads greater than 0.33 feet this value was sufficiently precise for practical work. The form of (52) is identical with our fundamental equation (43).

To correct for velocity of approach Bazin considered the effective head equal to $H + mh$ (m being an experimental coefficient); and by expressing h in terms of p, the height of the crest above the bottom of the channel, was able to obtain

$$Q = \left[0.405 + \frac{0.00984}{H}\right]\left[1 + 0.55\left(\frac{H}{p+H}\right)^2\right] b\sqrt{2g}\,H^{\frac{3}{2}}. \qquad (53)$$

The first bracket quantity is the coefficient c and the second is the velocity correction. For heads between 0.33 and 1.0 feet he states that we may use

$$Q=\left[0.425+0.212\left(\frac{H}{p+H}\right)^2\right] b\sqrt{2g}\, H^{\frac{3}{2}}, \quad . \quad . \quad . \quad (54)$$

with an error less than 3 per cent. The interesting fact concerning (53) is that it provides for the influence of the velocity of approach and allows Q to be calculated directly without resorting to the method of trial.

Bazin's weirs ranged from 1.64 to 6.56 feet in width, and the head from 0.164 to 1.969 feet. The value of p varied from 0.66 to 6.56 feet.

65. Formulæ of Hamilton Smith.—In 1886 Hamilton Smith, a noted American hydraulic engineer, published in his "Hydraulics" a very careful résumé and compilation of the best and most trustworthy experiments on weirs. From a study of the work of Francis, Fteley and Stearns, and others, he proposed for

Contracted Weirs.—

$$Q = c \times \tfrac{2}{3}\, b\sqrt{2g}\, H^{\frac{3}{2}}, \text{ with no vel. of approach.} \quad . \quad (55)$$

$$Q = c \times \tfrac{2}{3}\, b\sqrt{2g}\, (H + 1.4\, h)^{\frac{3}{2}}, \text{ with vel. of approach.} \quad (56)$$

and for

Suppressed Weirs.—

$$Q = c \times \tfrac{2}{3}\, b\sqrt{2g}\, H^{\frac{3}{2}}, \text{ with no vel. of approach.} \quad . \quad (57)$$

$$Q = c \times \tfrac{2}{3}\, b\sqrt{2g}\, (H + \tfrac{4}{3}\, h)^{\frac{3}{2}}, \text{ with vel. of approach.} \quad (58)$$

Formulæ (55) and (57) are identical with the fundamental equation (43), having values of c dependent on the type of weir and varying with its length and effective head $(H + mh)$. As for his mode of correcting the head to care for the velocity of approach, we see it is similar to that used by Fteley and Stearns. The difference between 1.4 h as used in (56) and $\frac{4}{3}\, h$ appearing in (58) can probably be explained by saying that a weir with two end contractions is generally placed near to the center of the channel where the water is moving with its highest velocity.

Tables VII and VIII give values for c for a wide range of conditions. The first column in each table represents the effective head $(H + mh)$, but in selecting a coefficient for use no appreciable error will result if we use H instead, since mh is very small. For heads and lengths not given c may be found by interpolation.

66. General Agreement of the Formulæ. — Although the formulæ of the different experimenters differ quite widely in the mode of correcting for contraction and velocity of approach, nevertheless there is a remarkable agreement in the quantitative results obtained by applying them severally to a particular weir problem. Gibson, in his "Hydraulics and its Applications," has given the following table, showing such results for the case of a suppressed weir 10 feet long, discharging with different values of head and velocity of approach.

Length.	Crest Height.	Head.	Velocity Approach.	Francis.	Fteley-Stearns.	Bazin.	Smith.
10 ft.	2 ft.	1.0 ft.	1.16 f. s.	1.00	1.015	.985	1.018
10 ft.	4 ft.	1.0 ft.	.68 f. s.	1.00	1.001	1.010	1.002
10 ft.	4 ft.	4.0 ft.	2.15 f. s.	1.00	1.050	1.050	1.115

For the purpose of comparison, the quantity as obtained by the Francis formula is taken as the "unit of comparison" and the quantities are expressed in terms of this unit. It will be noticed that, although Francis' formula gives slightly smaller results in this particular case, the general agreement of all the formulæ is very close.

67. General Observations. — It was particularly pointed out by Bazin as well as by Fteley and Stearns that the height of the weir crest above the bottom of the channel was an influential factor in determining the value of the velocity of approach and the amount of crest contraction. Francis, as a result of his observations, believed it desirable that this height be at least equal to $3\,H$ in order to obtain complete contraction. Since his coefficient was deduced from experiments where this condition was followed, it is essential that weirs to be gauged by his formulæ should be so constructed.

In the formulæ of Bazin and of Fteley and Stearns, provision was made for the varying of this dimension, but these engineers pointed out the desirability of keeping the water below the crest as deep as possible, since it reduced the velocity of approach and the probable error due to it. It has been shown experimentally (and can be proven mathematically) that the effect of the velocity of approach is negligible if the area of the cross-section of the channel be about 6 times the product of bH. For a suppressed weir this would be the equivalent of requiring the height of the weir crest to be 5 times the head upon it. Generally this proportion is not found in practice and a velocity correction is necessary. In the case of a weir with end contractions all experimenters state that, for accurate application of their formulæ, the end contractions must be made complete by allowing a clearance of at least $2H$ and preferably $3H$ between the ends of the weir and the sides of the channel. Using the latter figure, it can be shown that for values of b less than $10H$ the product of bH will be less than $\frac{1}{6}$ the area of the channel's section. Such a weir would require, therefore, no velocity correction, the probable error arising from its neglect being about $\frac{1}{2}$ of 1 per cent.

68. Submerged Weir.—If the water level on the downstream side of a weir be made to stand above the level of the crest, the weir is said to be *Submerged*. Using a weir with no end contractions, Fteley and Stearns experimented under the above

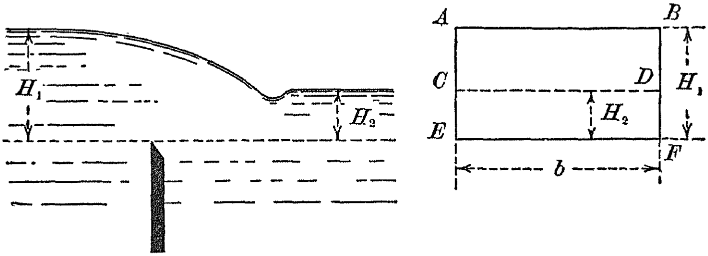


FIG. 75.

conditions to determine a rational formula for flow and coefficients for the same. This formula may be obtained by considering that the flow takes place in two parts,—one through

the weir *ABCD* (Fig. 75), and the other through the orifice *CDEF*. The theoretical discharge through the weir part would be

$$Q = \tfrac{2}{3} b \sqrt{2g}\, (H_1 - H_2)^{\frac{3}{2}},$$

and through the orifice part,

$$Q = bH_2 \sqrt{2g}\, (H_1 - H_2)^{\frac{1}{2}}.$$

By adding these, there results

$$Q = \frac{2}{3} b \sqrt{2g} \left(H_1 + \frac{H_2}{2}\right) (H_1 - H_2)^{\frac{1}{2}}.$$

Fteley and Stearns proposed

$$Q = cb \left(H_1 + \frac{H_2}{2}\right) \sqrt{H_1 - H_2} \quad . \quad . \quad . \quad . \quad . \quad (59)$$

c having a value dependent on $\dfrac{H_2}{H_1}$ as shown in the accompanying table: —

$H_2 \div H_1 =$	0.01	0.04	0.08	0.12	0.16	0.20	0.30
$c =$	3.33	3.34	3.37	3.35	3.32	3.29	3.21
$H_2 \div H_1 =$	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$c =$	3.15	3.11	3.09	3.09	3.12	3.19	3.36

Experiments on submerged weirs are too few to give accuracy and certainty to the results obtained by formulæ. It is difficult to measure the downstream head on account of the fluctuation of the water level at this point. Such weirs should therefore never be used for accurate measurements. It may happen that, during the gauging of a natural stream, an unforeseen rush of water may submerge the weir, and the above formula will then aid in a close approximation of the discharge during the flood interval.

✓ **69. Triangular Weir.** — Triangular weirs are sometimes used when the quantity of water flowing is not large. The customary arrangement is shown in Fig. 76, both sides of the notch being equally inclined from the vertical. Inasmuch as the issuing stream is of similar cross-section for all heads, the value

[illegible] Be able to develop equation (60) by calculus.

of the coefficient should be fairly constant. Experiment has shown this to be the case.

The formula for theoretical discharge may be obtained as follows. In Fig. 77, let x be the head on an elementary hori-

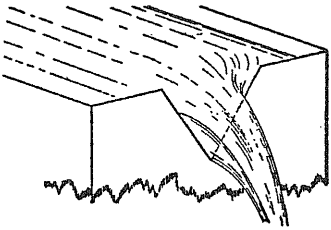


FIG. 76.

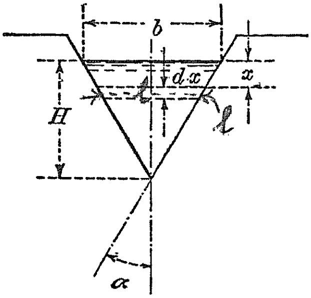


FIG. 77.

zontal strip. From similar triangles its length is $b(H-x) \div H$, = b
and for its area we have

$$dA = \frac{b}{H}(H-x)dx.$$

The discharge through the strip is

$$dQ = \frac{b}{H}(H-x)\,dx\,\sqrt{2gx} = \frac{b}{H}\sqrt{2g}\,(Hx^{\frac{1}{2}} - x^{\frac{3}{2}})dx,$$

and if this be integrated with H and 0 as the limits of x we obtain

$$Q = \tfrac{4}{15}\,b\sqrt{2g}\,H^{\frac{3}{2}}. \qquad (60)$$

The sides of the triangle being equally inclined, $b = 2H\tan\alpha$.

$$\therefore Q = \tfrac{8}{15}\tan\alpha\sqrt{2g}\,H^{\frac{5}{2}}.$$

Making the vertex 90° so that $\alpha = 45°$, we have

$$\text{Actual } Q = c \times \tfrac{8}{15}\sqrt{2g}\,H^{\frac{5}{2}}. \qquad (61)$$

For the value of c, Professor Thompson obtained 0.592 as a mean value when the range in head was from 0.2 to 0.8 feet. Using this value,

$$Q = 2.53\,H^{\frac{5}{2}}. \qquad (62)$$

Note:- b is distance across top of water here, and not across top of weir.

If a velocity of approach exists, H must be replaced by $(H+1.4\,h)$, as in the case of a rectangular weir with end contractions.

70. Trapezoidal Weir of Cippoletti. — This weir, invented by an Italian engineer whose name it bears, is mentioned because of its ingenious design, and the fact that it is more or less used in irrigation work. It has the advantage of a certain amount of constancy in its coefficient. As the name indicates, it has a notch of trapezoidal form as shown in Fig. 78. The side slopes are the same, and have an inclination of 1 in 4. The reason for this is of interest. The discharge may be divided into two parts, — one through the rectangular area of length b, and the other through a triangle having a base width of $2\,d$. The total discharge is therefore greater than from a rectangular contracted weir of length b. Cippoletti proposed giving the sides such a slope that this increase would be just equal to the decrease in discharge through a contracted weir, caused by end contractions. This would make the trapezoidal weir the equivalent of a rectangular suppressed weir of length b. The increase, being the discharge through the end triangles, may be written from equation (60), as

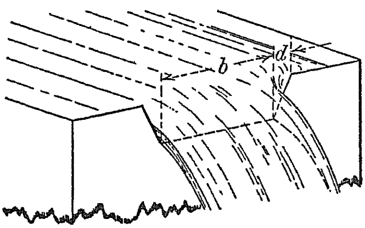


FIG. 78.

$$Q=c\times\tfrac{4}{15}\times 2\,d\sqrt{2g}\cdot H^{\frac{3}{2}},$$

and the decrease due to end contractions is, according to Francis,

$$Q_1=c\times\tfrac{2}{3}\sqrt{2g}\times 0.2\,H^{\frac{5}{2}}.\quad \text{(See equation 45.)}$$

Equating these values and assuming the two values of c to be alike, there results

$$d=\frac{H}{4},$$

giving the slope which Cippoletti recommended. From such a weir we may then figure Q by Francis' formula,

$$Q=3.33\,bH^{\frac{3}{2}}.$$

Cippoletti believed, as a result of experiment, that the value 3.33 was too small, and proposed

$$Q = 3.367\, bH^{\frac{3}{2}}.$$

Messrs. Flinn and Dyer in 1893 experimented with a weir of this type and obtained a value for the coefficient of 3.283. This was the mean value derived from 32 experiments. In correcting for the velocity of approach, however, they followed the method of Hamilton Smith (total head $= H + 1.4\, h$), while Cippoletti used the method of Francis. (See Art. 62.) The difference in method would nearly account for the difference in coefficient; and their conclusion was that the formula proposed by Cippoletti himself gave results within one per cent of the truth.

More experiments on this weir are desirable in order that our knowledge of the coefficient may be sufficiently precise to warrant its use as an efficient measuring device without previous calibration.

71. Water Measurement by Weirs. — The rectangular weir at present furnishes the most accurate method for measuring large quantities of water. Either the contracted or suppressed type may be used, although the latter is often preferred because of the absence of any uncertainty due to the end contractions. Certain precautions, some of which have already been enumerated, must be observed.

(1) The crest must be sharp-edged and horizontal; and its length should preferably be greater than $3\, H$.

(2) The plane of the weir's upstream face must be vertical. An upstream inclination *reduces*, and a downstream inclination *increases*, the discharge (Fig. 79). This results from the increased and decreased crest contraction. Bazin, in an account of his experiments, gives a discussion on the effect of inclination and also comparative values for the discharge as the inclination varies.

(3) The height of the crest above the bottom of the channel should be made as large as possible in order to reduce the velocity of approach. This is the same as saying that H should be made small in comparison with the depth of water in the channel.

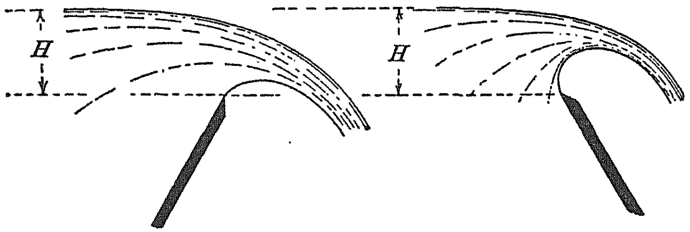


FIG. 79.

If possible, the area bH should not be larger than $\frac{1}{6}$ the area of the channel section.

(4) The end contractions, when present, should be made complete by observing a clearance of from 2 to 3 H between the ends of the weir and the sides of the channel.

(5) Provision must be made at the ends of the weir for the free admission of air to the space beneath the falling sheet or *nappe*. The existence of a partial vacuum here causes a depression of the *nappe*, and an increased discharge follows. (See Bazin's experiments.)

(6) The head H must be accurately measured. This can best be done by means of the hook gauge described in Art. 51, although for approximate work a graduated scale can be set vertically in the stream with its zero at the level of the crest. It has already been pointed out that the measurement must be made at a point far enough back from the weir to escape the effect of the surface curve. Fteley and Stearns recommend a distance equal to $2\frac{1}{2}$ times the height of the weir, but state that in a deep channel this distance may be made less.

Preferably the gauge should not be located too near the side of the channel in order to avoid the slight rise in the surface due to capillarity. As a position near the center of the stream would make the scale difficult to read, it is customary to place the gauge in a nearby pit or sump to which the water is led by a pipe connection. If the pipe be made to enter the weir channel at right angles to, and flush with, its side (see Art. 36 on piezometric readings), the water in the small pit will stand at the same level as in the channel. To ascertain the reading of the gauge when the point of the hook and the weir crest are

in the same horizontal plane, the water may be drawn down in the channel until it stands level with the crest. Some inaccuracies are apt to be met with, however, due to capillary action at the crest; and it is much better if a delicate spirit level be used to level a straightedge resting with one end on the crest and the other on the point of the hook.

As a result of experimental comparison Williams has concluded that it is of great importance, in measuring the head, that the same method be used as was employed by the experimenter whose formula is to be used. Otherwise an error of 10 per cent is liable to occur. References to the literature containing accounts of standard experiments are given at the end of this chapter.

If all the precautions enumerated above be observed, it is possible that the results obtained may be in error less than 2 per cent. Such accuracy, however, is not to be looked for in ordinary work and can be attained only by an experienced person.

72. Weirs with Rounded or Wide Crests. — Messrs. Fteley and Stearns experimented with weirs having the upstream edge of the crest slightly rounded. They found that the rounding increased the discharge by diminishing the contraction. The

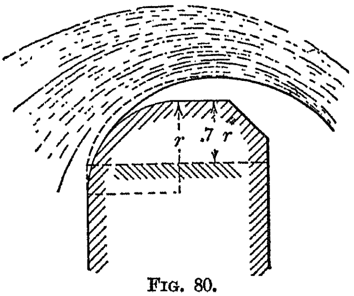


FIG. 80.

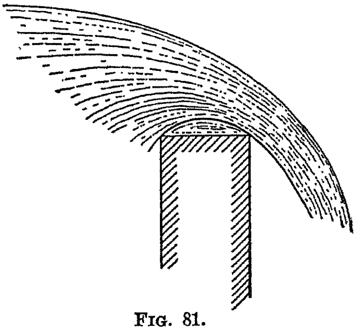


FIG. 81.

case is comparable with that of the orifice. When the rounding was made by a quarter circle having a radius of less than 0.5 inch, they found that the discharge could be determined by the usual formula, provided 0.7 the radius (in feet) be added to the measured head (Fig. 80).

If the crest be squared, but made so wide as to cause the water to touch again after leaving the inner edge, the amount discharged is usually diminished by the increase in friction. The experiments of Fteley and Stearns illustrate two possible cases. Figure 81 shows a crest just wide enough to slightly interfere with the descending sheet. The flow may be increased slightly if the space between the crest and the water contains a partial vacuum. If the crest be so wide as to resemble Fig. 82, the flow will be diminished by the increase in frictional resistance. Rounded or wide crests should be avoided in precise measurements.

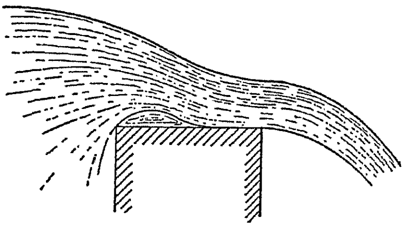


FIG. 82.

73. Dams used as Weirs. — The many dams existing on natural streams offer opportunities for gauging flow if their coefficients can be ascertained. As the shapes or profiles of dams vary greatly in detail, it would be a difficult and laborious task to determine and tabulate the coefficients for the many various types. However, two excellent sets of such experiments on dams have been recently completed. The first was by Bazin in 1897, carried on at the request of the French government. The second, in 1898, was under the direction of G. W. Rafter for the U. S. Deep Waterways Commission, carried on at the Cornell Hydraulic Laboratory. The results of both sets of experiments appear in a paper by Mr. Rafter found in the Transactions of the American Society of Civil Engineers, Vol. 44, 1900. The formula proposed is in the form

$$Q = c \times \tfrac{2}{3}\sqrt{2g}\, bH^{\frac{3}{2}} = MbH^{\frac{3}{2}}, \quad . \quad . \quad . \quad (63)$$

and values of M are given for a great variety of profiles and conditions.

PROBLEMS

1. A weir with end contractions has a crest 10.37 feet long and 2.87 ft. above the bottom of the channel. If the channel be 14.3 ft. wide, what amount will be discharged by the weir under a head of 0.875 ft.? Use both Francis' and Smith's formulæ, and compare results.

2. A suppressed weir, 6.97 ft. long, has its crest 2.79 ft. above the bottom of the channel. Compute the discharge for a head of 0.679 ft., using (*a*) Fteley and Stearns' formula; (*b*) Bazin's formula.

3. A rectangular channel 15 ft. wide contains water flowing 4 ft. deep, with a mean velocity of 2.2 ft. per second. If a suppressed weir, 4.5 ft. high, be built across the channel, how much will the level of the water back of it be raised? Use Francis' formula, (1) neglecting velocity of approach, (2) with velocity of approach.

4. A reservoir whose area is 12,000 sq. ft. has an outlet through a contracted weir whose crest is 3 ft. long. If, at the moment of opening, the head on the weir be 2.8 ft., find the time required to lower the surface 1 ft.

5. A reservoir 50 ft. by 200 ft. in plan, has its sides vertical. A rectangular weir is built in its side and stop planks prevent its discharging. When the water has risen to a height of 15 in. over the crest, the planks are removed. Find the length of the weir necessary to lower the reservoir 13.5 in. in 30 min. Use Francis' formula for suppressed weirs.

6. The angle of a triangular weir is 90°. How high must the water rise above the vertex so that the discharge may be 1000 gal. per min.?

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CHAPTER VIII

FLOW THROUGH PIPES

74. General Equation for Flow. — It will be convenient in approaching the subject of flow through pipes if we make the assumption of *steady, non-sinuous flow* which was outlined in Art. 30. By so doing, we shall greatly simplify our mathematical work and make it possible to derive a formula for flow which will have a rational basis and be fairly satisfactory in its application. It may then be corrected to agree with experimental results by applying coefficients or exponents to the several quantities involved.

The simplest case of flow occurs when a straight pipe of uniform section is inserted in the side of a reservoir and allowed to discharge under constant head into the atmosphere (Fig. 83).

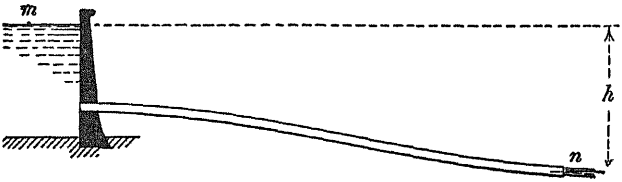

FIG. 83.

The flow being steady, we may write Bernoulli's Theorem between the points m and n, obtaining

$$0 + \frac{p_a}{w} + h = \frac{v^2}{2\,g} + \frac{p_a}{w} + 0 + \text{Lost head,}$$

or

$$h = \frac{v^2}{2\,g} + \text{Lost head.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

The last term covers all the losses in head occurring between the reservoir and outlet, no matter how caused. If each of these can be expressed as a function of the actual velocity, then

If you are given a pipe or hose ~~less~~ more than 1000 ft. long, don't consider entrance loss, because it becomes comparatively low

equation (64) will contain but one unknown, v, and may be used as a general equation for solving problems of pipe flow.

In a *straight* pipe of *uniform* section, the only losses in head to be considered are: —

(*a*) Head lost at entrance to the pipe.
(*b*) Loss due to internal friction.
(*c*) Loss due to friction between pipe and water.

The last two of these are so inextricably involved, one in the other, that no attempt will be made to separate them. They will be jointly considered under the caption of "Friction Loss."

If the pipe contains sudden changes either in direction or diameter, or if at any point the flow is hindered by the presence of partially closed gates or valves, these conditions will lead to further losses in head, the magnitude of which will now be discussed.

75. Loss at Entrance. — The conditions of flow in the first three diameters of the pipe's length are similar to those existing in a standard short tube which discharges against a head of water h_1 (Fig. 84). As there the velocity of flow is dependent on the available head $(h - h_1)$, so in the case of the pipe the velocity is dependent on the available head (h – friction head), as may be seen from a study of equation (64). The loss in the short tube may be obtained as follows: —

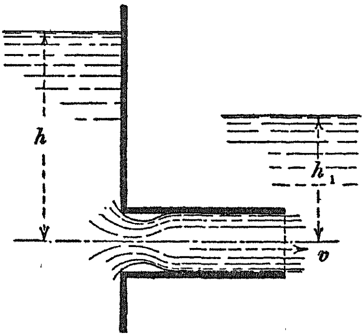

FIG. 84.

Referring to Art. 54, it will be seen that the head lost in any discharging device may be written: —

$$\text{Lost head} = \left(\frac{1}{c_v^2} - 1\right)\frac{v_a^2}{2g},$$

v being the actual velocity of flow. For the short tube $c_v = 0.82$, so that

$$\text{Lost head} = \left[\frac{1}{(0.82)^2} - 1\right]\frac{v^2}{2g} = 0.5\,\frac{v^2}{2g}.$$

Since this loss is expressed as a function of the *actual* velocity in the tube, it would seem as if we might further compare the tube with the pipe at entrance, and say that this same function of the velocity found there represents the loss at entrance. We will therefore assume,

$$\text{Loss at entrance} = 0.5\frac{v^2}{2g}. \quad . \quad . \quad . \quad . \qquad (65)$$

76. Friction Loss. — Of all the losses mentioned in Art. 74, the chief in magnitude (unless the pipe be short) is that due to friction along the inner wall and among the water particles. The law which governs its proportions, as the diameter of the pipe, the nature of its inner surface, and the velocity through it are made to change, has probably been the subject of more experiments and writing than any other field in Hydraulics. In spite of this fact, we have little exact knowledge of the law, and our formulæ which express it give only approximate results. Even temperature enters into the problem to add to its complexity. From the many experiments that have been made, we do know that the amount of frictional resistance offered is: —

(1) Independent of the pressure in the pipe.
(2) Proportional to the extent of frictional surface.
(3) Variable with the velocity of flow, and nearly proportional to the second power of the velocity, if the latter is above the "critical." (See Art 29.) If the velocity is below the critical, the resistance varies as the first power. (See Art. 78.)

Any formula, therefore, which is to properly express this loss should have some rational basis and recognize the above conditions.

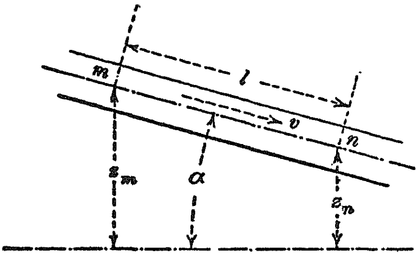

FIG. 85.

One treatment of the problem is as follows. The accompanying sketch represents a portion of the pipe shown in Fig. 83. The reservoir and pipe are supposed to be filled with water and the outlet stopped. If the

latter be opened, the action of gravity will cause the water to flow with increasing velocity until the resulting increase in friction between the water and pipe, and among the water particles themselves, causes a resistance that is capable of balancing the action of gravity. The accelerating force then becomes zero and steady flow ensues. In any cross-section of the pipe the mean velocity has then reached its maximum value. Across the section the velocity will vary from a maximum at the center to a minimum at the sides (Art. 88). We will neglect this variance, however, and consider all the particles as having the mean velocity v. Referring to Fig. 85, m and n are two points on the axis of the pipe at distances z_m and z_n above the datum. Between the two points we may write (Bernoulli's Theorem),

$$\frac{v_m^2}{2g} + \frac{p_m}{w} + z_m = \frac{v_n^2}{2g} + \frac{p_n}{w} + z_n + \text{Lost head},$$

and since the two velocities are equal (the pipes being of uniform section),

$$\text{Lost head} = \left(\frac{p_m}{w} + z_m\right) - \left(\frac{p_n}{w} + z_n\right). \quad . \quad . \quad . \quad (66)$$

To otherwise express the loss in head, consider the cylinder of water between sections m and n, and the forces acting on it (Fig. 86). Ap_m and Ap_n are the pressures exerted on the ends

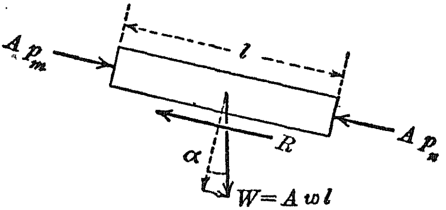

FIG. 86.

by the adjacent particles, A representing the area of cross-section. R is the total frictional resisting force, and Awl is the weight of the water. The motion being uniform, we may write

$$Ap_m + Awl \cdot \sin \alpha - R - Ap_n = 0,$$

or

$$\frac{p_m}{w} + l \sin \alpha - \frac{p_n}{w} = \frac{R}{Aw}.$$

From Fig. 85 we have

$$l \sin \alpha = z_m - z_n,$$

which inserted in the above gives

$$\left(\frac{p_m}{w} + z_m\right) - \left(\frac{p_n}{w} + z_n\right) = \frac{R}{Aw},$$

and this combined with (66) results in

$$\text{Lost head} = \frac{R}{Aw}.$$

If now the frictional resistance per unit area of pipe surface be F, so that $R = \pi d l F$, we may write

$$\text{Lost head} = \frac{4\,\pi d\,lF}{w\pi d^2} = \frac{4\,lF}{wd}. \quad . \quad . \quad . \quad (67)$$

In the first of the present paragraph it was pointed out that we have no exact knowledge of the value of F; hence at this point fairly rational treatment of the problem ceases, and some assumption regarding F must be made. Since for velocities above the critical, F appears to vary nearly as the second power of the velocity, we may write

$$F = cv^2,$$

obtaining

$$\text{Lost head} = \frac{4c}{w}\,\frac{l}{d}\,v^2.$$

If we replace $\frac{4c}{w}$ by f, there results

$$\text{Lost head} = f\frac{l}{d}v^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

The nature of the quantity f may be learned from the following interpretation of equation (68).

$$\text{Distance} = f \times \frac{\text{distance}}{\text{distance}} \times \left(\frac{\text{distance}}{\text{time}}\right)^2.$$

$$f = \frac{(\text{time})^2}{\text{distance}}.$$

We see that it is a function of both *time* and *distance*, and will therefore vary in value according to the system of units employed in measuring these quantities. It would be much better if f were an abstract number, and it may be made so by introducing into the right-hand member of (68) a quantity in the form of $\frac{(\text{time})^2}{\text{distance}}$. The quantity $\frac{1}{2g}$ may be shown to have this form, and its introduction into the equation secures the added advantage of permitting the lost head to be expressed as a function of the velocity head. Equation (68) may then be written

$$\text{Lost head} = f\frac{l}{d}\frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad (69)$$ ✓

This formula is probably more widely known and used than any other. Later it will be shown to be identical with the formula of De Chezy for flow through pipes.

77. Determination of f. — The quantity f is generally spoken of as the *Friction factor* and its value for any particular pipe and velocity of flow may be obtained experimentally as follows: —

Figure 87 shows two piezometers inserted in a pipe at two sections a distance l apart. Since the velocity at both sections is the same (diameter constant), we may write,

$$\frac{p_m}{w} + a = \frac{p_n}{w} + \text{Lost head},$$

and

$$\text{Lost head} = \frac{p_m}{w} - \frac{p_n}{w} + a = h.$$

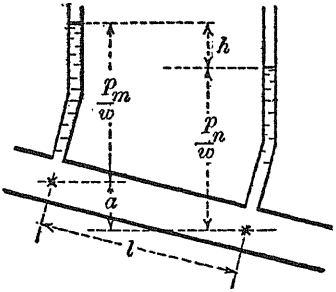

FIG. 87.

By measuring h, a, and Q the value of the lost head is found, and (69) may be solved for f.

As a result of many experiments it has been found that

(1) f decreases, in the same pipe, as velocity increases.

(2) f decreases, with the same velocity, as the diameter increases.

(3) f varies *largely* with the condition of the interior surface of the pipe, being larger, the rougher the surface.

$f = .0199 + \frac{.00166}{d}$ is a good one for pipes to us
([illegible] practical exhts.).

In view of the last fact it will be seen that it is difficult to estimate a value of f for any particular pipe, since even in pipes of the same material the roughness of the lining may vary. Much more difficult and quite impossible is it to predict what value of f will hold for a pipe after it has become coated with rust or other deposits as the result of long use. Determinations of f have been made for a number of materials, including cast-iron, wood, lead, brass, glass, wrought-iron and riveted steel. Table IX of the Appendix shows probable values of f for *clean* cast-iron pipes, based on data given by Fanning in his "Treatise on Hydraulic and Water Supply Engineering." An inspection of this table shows that a mean value is given by the decimal 0.02. Since f varies with v, any problem in which v is unknown must be solved by first using a trial value for f, thus allowing v to be found approximately. This value of v then serves to find a closer value for f, which in turn is used to obtain a second value for v. Generally this latter will indicate a value of f so close to that last assumed that further solutions are unnecessary.

Example. — A straight 12-inch pipe, 4000 feet long, is connected with a reservoir containing water whose level is 100 feet above the discharging end. Find the probable velocity of flow.

From equation (64),

Entrance ~~loss~~ head   friction head

$$h = \frac{v^2}{2g} + 0.5\frac{v^2}{2g} + f\frac{l}{d}\frac{v^2}{2g}.$$

$$100 = \frac{v^2}{64.4} + \frac{0.5\,v^2}{64.4} + \frac{0.02 \times 4000}{1}\frac{v^2}{64.4}.$$

$$v = 8.9 \text{ feet per second.}$$

From Table IX, f corresponding is 0.0203.
Using this value, we find,

$$v = 8.8 \text{ feet per second.}$$

The corresponding value of f remains 0.0203 and no further solutions are necessary. For all practical purposes the result may be stated as 9.0 ft. per second since the uncertainty in the value of f does not warrant greater precision.

An idea of the relative size of the friction loss may be obtained from the following summary.

Velocity head =	$\frac{v^2}{2g} =$	1.21 ft.
Entrance loss =	$0.5\frac{v^2}{2g} =$	0.60 ft.
Friction loss =	$f\frac{l}{d}\frac{v^2}{2g} =$	98.19 ft.
	Total	100.00 ft.

It is seen that 98 per cent of the available head was lost in pipe friction.

78. Relation between Frictional Resistance and Velocity. — For any one pipe the amount of frictional resistance must be some function of the velocity. We have so far assumed it to vary as v^2; but experimental work has shown that in general the value of the exponent is somewhat less than 2. The general relation between lost head and velocity may be expressed by the equation

$$h_l = kv^n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

both k and n being constant for any particular pipe. The equation may be written

$$\log h_l = \log k + n \log v, \quad . \quad . \quad . \quad . \quad (71)$$

which is the equation of a straight line, since it is in the form of

$$y = a + nx.$$

If now we experimentally determine corresponding values of v and h_l for a wide range of velocities, and plot the logarithms of these quantities as abscissæ and ordinates respectively, we should expect to obtain, as their locus, a straight line making some angle α with the axis of $\log v$, and cutting off an intercept on the axis of $\log h_l$ equal to $\log k$. The value of n would then be obtained from $\tan \alpha = n$.

Figure 88 shows such a plot as made by Professor Reynolds from data obtained by experiments on a small lead pipe. As

the velocities were increased from very small values, Professor Reynolds found that the plotted points, up to a certain

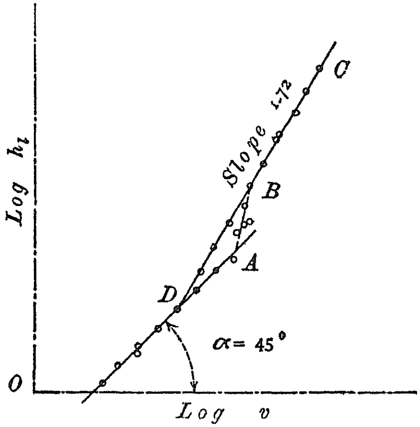

FIG. 88.

point A, lay on a straight line which made an angle of 45° with the log v axis. The value of n for this line being unity, and that of log k zero, it will be seen that for low velocities the loss in head was directly proportional to v. At A the position of the points indicated a sudden increase in loss, but no definite law governing the change. Not until a point B was reached and passed did the law become apparent, and then it was found that further increase in v caused all the points to lie on a straight line BC. This line had such a slope that tan α equaled 1.722, showing that h_l varied as $v^{1.722}$. The point A marks the *higher* critical velocity at which the *non-sinuous* motion, which the water previously had, changed to *sinuous* motion. Reynolds found also that by starting with high velocities and gradually decreasing them, the plotted points followed the line BC down to a point D, the intersection of BC and AO. This point marks the *lower* critical velocity, at which motion, that had previously been sinuous, becomes non-sinuous. This lower value of the critical velocity is the one we are concerned with in pipe flow, inasmuch as water almost invariably enters the pipe in a turbulent state.

It was pointed out in Art. 29 that the value of the critical

velocity was constant for any particular pipe, varying only with the temperature of the water. With temperature constant, in different pipes it becomes a function of the diameter. As the result of his experiments, Reynolds proposed as a value for the critical velocity,

$$v_c = \frac{0.0388\,P}{d},$$

the value of P being

$$P = \frac{1}{1 + 0.0336\,t + .0000221\,t^2},$$

and d being the diameter in feet.

By t is represented the temperature of the water in degrees centigrade. It will be seen from the formula that the value of the critical velocity is very small, too small in general to interest the engineer. The flow of water through soils and filter drains illustrates cases where the critical velocity has its importance.

79. Loss by Bends. — If water that is flowing in a straight pipe be suddenly deflected from its course by a bend, a loss in head follows, which is due to eddyings and impact. Our knowledge concerning the magnitude of this loss, as the radius of the bend, the size of pipe, and the velocity of flow varies, is at present very incomplete. Weisbach describes some experiments made by him on smooth iron pipes, $1\frac{1}{4}$ inches in diameter, bent in quarter circles. For the head lost at the bend, in excess of that occasioned by pipe friction, he proposes

$$\text{Lost head} = c\frac{v^2}{2g}, \quad \ldots\ldots \quad (72)$$

and gives the following table for c, in which d is the diameter of the pipe, and r the radius of the bend: —

For $\frac{d}{r} =$	.20	.40	.60	.80	1.0	1.2	1.4	1.6	1.8	2.0
$c =$	.131	.138	.158	.206	.294	.440	.661	.977	1.40	1.98

As the diameter and radii of bends were both small, and the number of experiments not numerous, it would appear that little reliance should be placed on the values of the coefficients, particularly when applied to pipes and bends of large size.

Notwithstanding this fact, Weisbach's formula and coefficients have long been used by engineers.

More recent than those of Weisbach are the experiments of Brightmore, Schoder, and Williams, Hubbell, and Fenkell. The last three experimented in 1893–98 with the cast-iron water mains of Detroit, Michigan, and measured the losses of head in bends of 90°. The sizes of pipe used were 12, 16, and 30 inches in diameter. The most remarkable thing in their conclusions was that, while Weisbach's experiments indicated an *increase* in loss with *increased* sharpness of bends, the Detroit experiments showed a *decrease* in loss as the bends sharpened, down to a limiting radius of about $2\frac{1}{2}$ diameters. Beyond this point there appeared to be a gradual increase. This conclusion, being so diametrically opposed to generally accepted principles, aroused keen interest and much discussion among hydraulicians. In view of the fact that the Detroit experiments were conducted in a most careful and scientific manner, much weight must necessarily be given them until further experiments under like conditions give reason to do otherwise. More recently (May, 1908), however, Professor Schoder of Cornell University has published the results of his experiments on bends of 90° in 6-inch pipes, and his conclusions substantiate the old theory that increased loss follows increased sharpness in bends. Mr. Brightmore, Member Inst. C. E., has also published (1907) some experiments on 3- and 4-inch pipes which quite agree in qualitative results with those of Professor Schoder. The weight of testimony therefore appears to favor the old theory. In the Detroit pipe line the bends were made by placing together short sections, giving rise to losses due to joints. The velocities were comparatively low and the measured losses therefore small. Schoder points out that this fact would tend to magnify excess losses due to other causes than curvature, thereby making it difficult to distinguish the true varying of the latter loss. In view of our imperfect knowledge of the subject and the fact that there were present in the Detroit experiments certain conditions whose influence was doubtful, it would seem best to continue for the present on the basis of the older theory and await more experiments.

One very important fact should be pointed out, and that is that the loss in head due to a bend does not occur entirely within the bend itself, but occurs also in the subsequent straight section where the eddyings caused at the bend gradually die out. This fact should be recognized in any experiments to determine the effect of curvature; and in placing the piezometers for measuring the total loss between two sections including the bend, the downstream one should be placed far enough below the bend to insure the perfect resumption of normal flow. In his 6-inch pipes Schoder anticipated that the influence of the bends might extend a distance of 100 diameters downstream, but subsequently found that conditions were normal at a distance of 76 diameters.

It is quite fortunate that, in long pipe lines having few bends, the loss in head from bends is inconsiderable when compared to the friction loss in the straight part. In bends having a radius of from $1\frac{1}{3}$ to 20 diameters, and velocities ranging from 3 to 16 feet per second, Schoder gives the excess loss in head in a 90°-bend over the head lost in an equal length of straight pipe, as equal to the loss in a straight pipe of the same size having a length of from 2 to 20 diameters and the same velocity of flow.

80. Loss by Sudden Enlargement of Section. — If the cross-section of the pipe be abruptly enlarged, as in Fig. 89, the velocity will be reduced from v to v_1, and a loss in head will result from the impact and eddying caused by the meeting of the more swiftly moving water in the small pipe with the slower water in the large pipe. To properly estimate the loss we may proceed as follows: Let the area of the cross-section in the pipe be a and a_1, respectively, so that $av = a_1v_1$. Writing Bernoulli's Theorem between m and n, we obtain

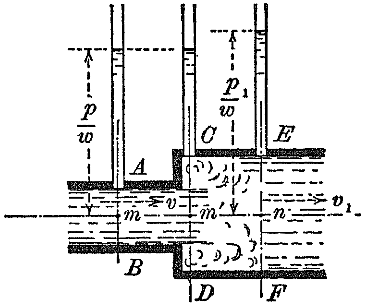

FIG. 89.

$$\frac{v^2}{2g}+\frac{p}{w}=\frac{v_1^2}{2g}+\frac{p_1}{w}+\text{Lost head},$$

from which

$$\text{Lost head}=\frac{v^2}{2g}-\frac{v_1^2}{2g}-\left(\frac{p_1}{w}-\frac{p}{w}\right). \quad . \quad . \quad . \quad (73)$$

The velocity v is maintained up to and slightly beyond $C\text{-}D$, so that at m' the pressure remains p. The total pressure in the large pipe on section $C\text{-}D$ must then be a_1p, while on $E\text{-}F$ it is a_1p_1. That p_1 is greater than p may be seen from the fact that between m and n there occurs a large decrease in velocity head without a corresponding gain in potential head. Consequently $\frac{p_1}{w}$ must be greater than $\frac{p}{w}$. There exists, therefore, an unbalanced pressure between $C\text{-}D$ and $E\text{-}F$ against which W pounds of water move each second and thereby have their velocity changed from v to v_1. Since a force is proportional to the change of momentum which it can produce in a given mass in a second's time, we may write,

$$a_1p_1-a_1p=\frac{W}{g}(v-v_1),$$

or

$$\frac{p_1}{w}-\frac{p}{w}=\frac{v_1}{g}(v-v_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Combining (73) and (74),

$$\text{Lost head}=\frac{v^2}{2g}-\frac{v_1^2}{2g}-\frac{v_1}{g}(v-v_1)=\frac{(v-v_1)^2}{2g}. \quad . \quad (75) \quad \surd$$

Another form of expression may be derived from the relation $av=a_1v_1$. Substituting in (75) for v its equivalent $\frac{a_1v_1}{a}$, there results

$$\text{Lost head}=\frac{\left(\frac{a_1v_1}{a}-v_1\right)^2}{2g}=\left(\frac{a_1}{a}-1\right)^2\frac{v_1^2}{2g}. \quad . \quad . \quad . \quad (76) \quad \surd$$

The validity of our reasoning has been shown in a set of experiments by Gibson, in which he obtained results in close accordance with those given by (75).

The difference in pressures existing in the two sections of the pipe may be found by substituting the value of the loss, as given by (75), in equation (73), and solving for $\frac{p_1}{w} - \frac{p}{w}$. There results,

$$\frac{p_1}{w} - \frac{p}{w} = \frac{v_1(v - v_1)}{g} \quad \ldots \ldots \quad (77)$$

81. Loss by Sudden Contraction in Section. — If a pipe suffers a sudden contraction of section in the direction of the flow within, a loss of head results. Reference to Fig. 90 shows it to be divided into two parts. The vertical shoulder AB causes impact and eddying in the end of the large pipe, and the water as it passes the sharp edge ab contracts as from an orifice, and subsequently suffers a loss of head in expanding to fill the pipe. The size of this loss will depend upon the ratio of the velocities and the amount of contraction at m, which is the same as saying it varies with the ratio of A_1 to A, A_1 and A being the respective areas of the small and large pipe. While a value may be approximated by treating the loss as due wholly to the sudden expansion beyond m, a preferable method is to express the loss as

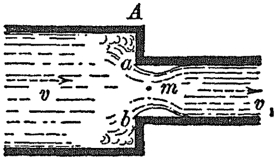

FIG. 90.

$$\text{Lost head} = k\frac{v_1^2}{2g}, \quad \ldots \ldots \quad (78)$$

and determine by experiment values of k for different ratios of A_1 to A. Professor Hoskins, in his recent (1906) work on Hydraulics, gives the following convenient table which is based on data from Weisbach: —

For $\frac{A_1}{A}$ =	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00
k =	.362	.338	.308	.267	.221	.164	.105	.053	.015	.0

82. Summary of Losses in Pipes. — For ready reference the different losses and their values are here summarized: —

(1) Head lost at entrance $= 0.5\dfrac{v^2}{2g}$.

(2) Head lost by pipe friction $= f\dfrac{l}{d}\dfrac{v^2}{2g}$.

(3) Head lost by bends $= c\dfrac{v^2}{2g}$.

(4) Head lost by sudden enlargement $= \dfrac{(v-v_1)^2}{2g}$

$$= \left(\frac{a_1}{a}-1\right)^2 \frac{v_1^2}{2g}.$$

(5) Head lost by sudden contraction $= k\dfrac{v^2}{2g}$.

If a pipe be in any way obstructed, the resulting loss may be approximately determined from (4) by letting a_1 represent the area of the pipe and a the area at the point of obstruction.

The general equation (64) may be now thus elaborated: —

$$h = \frac{v^2}{2g} + \frac{0.5\,v^2}{2g} + f\frac{l}{d}\frac{v^2}{2g} + \text{Losses by bends, etc.,} \quad . \quad (79)$$

and if the pipe be fairly straight and of uniform section throughout,

$$h = \frac{v^2}{2g} + \frac{0.5\,v^2}{2g} + f\frac{l}{d}\frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad (80)$$

In determining the velocity in a very long pipe it will be found that the loss by friction is so nearly the entire head h that we may write

$$h = f\frac{l}{d}\frac{v^2}{2g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

without an error greater than that liable to result in the selection of a value for f.

83. Solution of Pipe Problems. — In the above equations, the principal variables are h, l, d, and v. A majority of the pipe problems arising in practice consist in having one of these unknown and desired. The solution may be effected by using either (79), (80), or (81), as the case requires, and solving directly for the unknown. Two very common problems are:

(1) Given h, l, and d, to find v and Q. Choosing (80) as representative, we have,

$$v = \sqrt{\frac{2gh}{1.5 + f\frac{l}{d}}}, \qquad \text{and} \quad Q = Av, \quad . \quad . \quad (82)$$

where A, the area of the pipe, $= \frac{\pi d^2}{4}$.

(2) Given h, l, and Q, to find d necessary to deliver Q.

Again, using (80), and remembering that $v = \frac{4Q}{\pi d^2}$, we have

$$d = \sqrt[5]{\frac{1.5\,d + fl}{2\,gh}\left(\frac{4\,Q}{\pi}\right)^2}. \quad . \quad . \quad . \quad . \quad (83)$$

In Problem 1 a tentative value of 0.02 is first assumed for f, and then corrected to accord with the resulting value of v. The process is repeated until a value for v is obtained which requires no new value for f.

In Problem 2, f is taken again as 0.02, and since d appears under the radical sign, its value in this place must be tentatively assumed. From the first solution of (83) a tentative value of v may be found from

$$v = \frac{4\,Q}{\pi d^2},$$

and the corresponding f determined. With these two approximate values of d and f, we may solve (83) for a closer value of d, and so proceed until a satisfactory determination is made.

84. Hydraulic Gradient.—Figure 91 shows an open piezometer inserted at n, in a straight pipe of uniform section which discharges into the air. Between m and n we may write

$$0 + \frac{p_a}{w} + h_1 = \frac{v^2}{2g} + \left(\frac{p}{w} + \frac{p_a}{w}\right) + 0 + \text{Lost head},$$

$\frac{p}{w}$ being the height of the piezometer column above n, and h_1 the distance of n below the reservoir level. Evidently p is in

relative units, and would be zero if n were at the outlet of the pipe. (See Fig. 83 and equation (64).) More simply may the above be written

$$h_1 = \frac{v^2}{2g} + \frac{p}{w} + \text{Lost head}, \quad . \quad . \quad . \quad . \quad (84)$$

and it should be noted that h_1 is divided into three parts. Since one of these, $\frac{p}{w}$, is the height of the piezometer column, the sum of the other two $\left(\frac{v^2}{2g} + \text{Lost head}\right)$ is the distance down from the reservoir level to the top of the column. At any point below n, such as n', the velocity head will be unchanged, but the lost head will be increased by reason of the loss occurring in the intervening distance. Hence the sum of the two heads is larger than at n, and the level of the second column is lower than the first. The difference in level being e, we may write,

$$e = f\frac{l'}{d}\frac{v^2}{2g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

from which, if e and l' be measured, the value of v may be computed. A tentative value of 0.02 must be used in first solving for v, and may then be corrected and recorrected, as explained in Art. 77.

The equation also shows that in a straight pipe of constant section, the loss in head between two points, and consequently the drop in piezometer columns, is proportional to the length of the intervening pipe. Therefore, if a row of piezometers were inserted along the pipe from reservoir to outlet, a straight

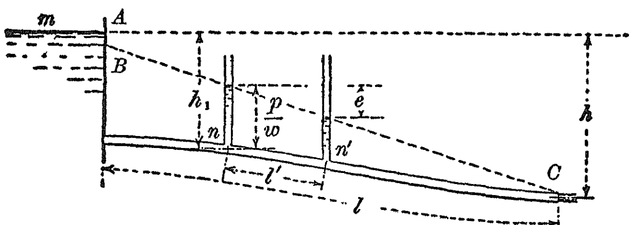

FIG. 91.

line would join the levels of their water columns. It would start (practically) from a point B (Fig. 91) a little below the

reservoir level, and run to C at the outlet. The distance AB represents the sum of the velocity head and the head lost at entrance; and the line must run to C, since the pressure there is only atmospheric. This line is commonly known as the *Hydraulic Grade Line* or *Gradient*, and a vertical ordinate between any point in the pipe line and gradient will measure the pressure head at that point. If the pipe have a considerable length, the slope of the gradient may be approximately written

$$s = \frac{h}{l}, \quad \ldots \ldots \ldots \quad (86)$$

and slight changes in vertical elevation may be made by gentle curves without sensibly changing the gradient from a straight line. (This follows from the fact that the length l would still remain practically the same as its horizontal projection.)

If a pipe be laid so that a portion of it comes above the gradient, a serious disturbance *may* result in the flow. At any point n (Fig. 92), in the part lying above the gradient,

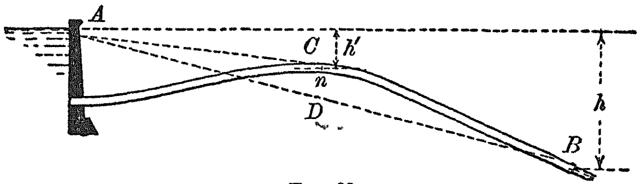

FIG. 92.

the pressure must be less than atmospheric, since the vertical ordinate is measured *downward* to the gradient. If the pipe be absolutely air-tight, flow will take place as in the ordinary case. If air can gain entrance to the pipe by joints or other means, then it will collect at the summit near n and the pressure will approach the *atmospheric*. If this condition of pressure be obtained, the gradient will shift to A-C and the discharge will be due to the head h'. The portion of the pipe between C and B will act as a channel to carry off the decreased flow from C.

The importance of knowing the relative positions of the gradient and the pipe line is at once seen. No long pipe line

having a small total fall and sensible variations in level should be laid in the ground until a gradient is plotted on the profile of its course and the absence of "summits" above the gradient assured. In pipes where summits do exist, full flow may be maintained by placing pumps at these points which shall exhaust the collected air at intervals during the period that flow must be maintained.

The theoretical limit to the distance *CD* which the pipe may rise above the gradient is $\frac{p_a}{w}$, or approximately 34 feet. The practical limit is much less. If the distance be made greater, the pressure tends to become less than zero and steady flow ceases.

85. Other Formulæ for Flow in Pipes. — *Chezy's Formula.* For the case of a long pipe, free from losses other than caused by pipe friction, we have seen that the law of flow may be fairly well expressed by the formula

$$h = f\frac{l}{d}\frac{v^2}{2g}.$$

This is commonly referred to as the formula of Chezy, although it differs somewhat in form from that proposed by him. The latter may be obtained as follows: —

The value of the ratio of the cross-sectional area of a pipe to the length of its perimeter is often referred to as its *hydraulic radius* or *hydraulic mean depth.* Thus if A be the area, and p the perimeter, we have

$$\text{Hydraulic radius} = r = \frac{A}{p}.$$

For a circular pipe flowing full, r will be found equal to $\frac{d}{4}$, and d in the above formula may be replaced by $4r$. (The term *hydraulic radius* or *hydraulic mean depth* has little significance in itself. It should be regarded simply as a name for an oft-recurring ratio. If the mass of water present in a pipe be imagined as spread out over a plane surface whose area is equal to that of the pipe's walls, then the depth of the water will be the same as the hydraulic mean depth of the pipe.)

By simple transposition the equation now becomes

$$v = \sqrt{\frac{8g}{f}} \times \sqrt{r\frac{h}{l}},$$

and if we replace $\sqrt{\frac{8g}{f}}$ by C and $\frac{h}{l}$ by s (see Art. 84), we obtain the formula as proposed by Chezy,—

$$v = C\sqrt{rs}. \qquad (87)$$

Since C has the value $\sqrt{\frac{8g}{f}}$, it should vary according to the same laws as f and depend on the velocity, the diameter, and the roughness of the pipe's lining. Experiment has shown such to be the case. In using equation (87) values of C may be obtained from Table IX and the relation

$$C = \sqrt{\frac{8g}{f}}. \qquad (88)$$

Exponential Formulæ. Not only has experimental work shown conclusively that the frictional resistance does not vary exactly with the second power of v, but it has also demonstrated that the resistance varies inversely as some power of diameter, but not the *first.* It would therefore appear that a formula in the form of

$$h = k\frac{lv^n}{d^x} \qquad (89)$$

is more desirable, since for a given pipe (roughness of lining and diameter fixed) the value of k would be constant, not varying with the velocity. To show the values which k, n, and x may have for a particular pipe, Lea cites some experiments on smooth brass pipes, from which he deduced,

$$h = 0.00296\frac{lv^{1.75}}{d^{1.25}}.$$

The sizes of the pipes were all under 2 in., and the exponent of v varied in them from 1.73 to 1.77, so that 1.75 represents a

mean value. It is perhaps unnecessary to state that these values for k, n, and x hold only for the pipes experimented upon, or their duplicates. Experiment has shown that both k and n vary with the roughness of the pipe's interior surface, and that x will vary slightly with both surface and diameter. Thus Professor Unwin derived

$$h = 0.0004 \frac{lv^{1.87}}{d^{1.4}}$$

for smooth cast-iron pipes, and

$$h = 0.0007 \frac{lv^2}{d^{1.1}}$$

for pipes with rough interior surfaces. The constants and exponents must be regarded, however, as *mean* values only, determined by averaging those obtained from many experiments on different pipes. Flamant, Lawton, Reynolds, and others have proposed formulæ very similar to (89), and furnished tables from which the value of the constants and exponents may be obtained for varying conditions. The point for the student to grasp here is that formulæ in this form are probably the most satisfactory yet devised, provided we can determine the values of k, n, and x proper for the problem at hand. This is a difficult thing to do, since it is impossible to strictly compare the pipe in question with those experimented upon, and say which one it most closely resembles. The wide variance among the constants and exponents which have been determined by many experimenters is well shown in the following formula, which is taken from Lea's excellent and recent (1908) work on Hydraulics : —

$$h = \frac{0.00028 \text{ to } 0.00069\ v^{1.70 \text{ to } 2.08}\ l}{d^{1.25}} \quad . \quad . \quad . \quad . \quad . \quad (90)$$

The formula was obtained from a logarithmic plotting of many experiments on pipes of different sizes and materials ; and the range in values of k and n show the wide variance as found in these quantities in the different experiments. Evidently the formula has no practical use beyond serving to illustrate this variance.

One serious disadvantage accompanying the use of these "exponential" formulæ is their cumbersomeness, their solution requiring the use of logarithms. While it is true that they more accurately express the laws of flow than does the simpler Chezy formula, the uncertainty attending the selection of proper values for the constants and exponents results in errors probably as large as found in the Chezy formula.

Hazen and Williams have proposed an exponential formula, based on Chezy's in its original form, which for practical purposes appears to be the best yet devised. They write

$$v = Cr^{0.63}s^{0.54}\,0.001^{-0.04}, \quad . \quad . \quad . \quad . \quad . \quad (91)$$

and the particular advantage which this formula offers is that C is very nearly constant in value for a fixed degree of roughness. It was pointed out in the first of this article that C in the Chezy formula varied in value, not only with the roughness of the surface, but with the velocity, the diameter, and the hydraulic slope. If the exponents of r and s could be given values which were in agreement with facts, then C ought to vary only with the degree of roughness. It is quite impossible to do this, however, since the exponents are found to vary with the surface, the diameter, and the slope. By using values, however, which represent the average conditions, the value of C for a given surface will vary so little as to be practically constant. The exponents used by Hazen and Williams were obtained from a careful study of the best experiments on such kinds and sizes of pipes as are commonly found in water works construction. The last term of their formula, $.001^{-0.04}$, is a constant introduced simply to make the value of C the same as in the Chezy formula.

The formula is unwieldy and would be difficult to use were it not that its authors have devised a slide rule which makes a solution easy and rapid. They also publish a set of "Hydraulic Tables" in which are given values of C for a wide range of conditions, and also convenient tables from which v and Q may be quickly taken for assigned values of d and s.

86. The Venturi Meter. — This device, invented by Herschel in 1887 and named by him after the distinguished philosopher

who experimented with diverging tubes, has for its object the measurement of the rate of flow in pipe lines. It is extremely simple in form and detail, consisting merely of two frustums of conical tubes joined by a short cylindrical section and inserted in the pipe whose flow is to be metered (Figs. 93 and 94).

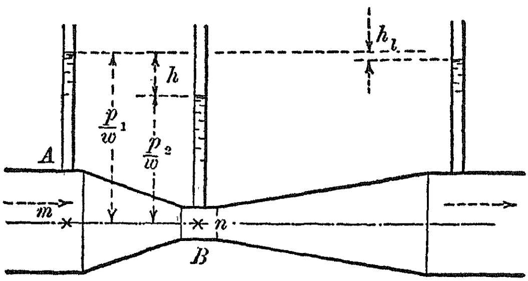

FIG. 93.

Pressure gauges, fitted to the main pipe at A and to the "throat" of the meter at B, measure the pressure heads at these

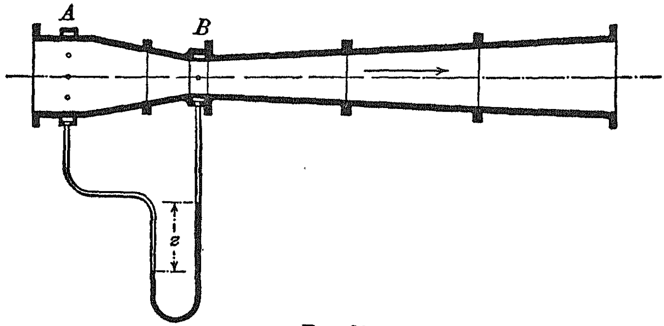

FIG. 94.

points and complete the apparatus in its essentials. Flow takes place in the direction indicated by the arrows, and the theory of its operation is as follows: Between the points m and n (the pipe being horizontal), Bernoulli's Theorem shows that

$$\frac{v_1^2}{2g}+\frac{p_1}{w}=\frac{v_2^2}{2g}+\frac{p_2}{w},$$

provided no loss in head occurs between the two points. If a_1

and a_2 represent the areas of the sections which contain the points, we may also write

$$Q = a_1 v_1 = a_2 v_2,$$

from which
$$v_1 = \frac{Q}{a_1} \text{ and } v_2 = \frac{Q}{a_2}.$$

These latter values substituted in our first equation give

$$Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2g\left(\frac{p_1}{w} - \frac{p_2}{w}\right)}. \quad . \quad . \quad . \quad (92)$$

To allow for the loss by friction occurring in the converging section, it is customary to introduce a coefficient c in equation (92) and determine its value by experiment. For any particular meter the equation may then be written

$$Q = k\sqrt{\frac{p_1}{w} - \frac{p_2}{w}}, \quad . \quad . \quad . \quad . \quad . \quad (93)$$

or $Q_t = a_2 v_2 = a_2 \sqrt{\frac{2gh}{1-(\frac{a_2}{a_1})^2}}$

k being assumed a constant for the meter. For large meters experiment indicates that this assumption involves little error. With small meters passing small quantities of water, k varies with the velocity.

Figure 94 shows a sectional view of a Venturi meter as commonly constructed. The converging portion is comparatively short, its length varying from 2 to $2\frac{1}{2}$ times the diameter of the main pipe. In this portion the loss is therefore very small, depending on the diameter, the material of the meter, and the velocity of flow. The throat of the meter usually has a diameter $\frac{1}{3}$ that of the main pipe, so that the velocity through it is 9 times the pipe velocity. The diverging portion is made *gradually expanding* in order that the loss in head due to decrease in velocity may be small. Experiment has shown that, for a given head, the discharge from a diverging tube will be a maximum when the angle of divergence is about 5°. Herschel reasoned, therefore, that a tube having this flare would cause the least loss in head for a given discharge, and designed his meter accordingly.

That the value of the coefficient c is fairly constant for large

$$Q_t = a_2 v_2 = a_2 \sqrt{\frac{2gh}{1-\left(\frac{a_2}{a_1}\right)^2}} = a_2 \sqrt{\frac{2\ldots}{1-\ldots}}$$

meters may be seen from the results of Herschel's experiments on meters fitted to 12, 48, and 108 inch pipes. Within the limit of ordinary use he found the coefficient ranged between 0.94 and 1.00, and a majority of the values were found to lie between 0.97 and 0.99. Professor Coker, in some similar experiments on a small meter having a throat area of 0.0144 sq. ft., found that for small flows the coefficient was very variable, ranging from 0.95 to 1.36. It is probable that for low velocities through small meters the discharge is not proportional to the square root of the head, as equation (92) would indicate. Hence the variance in the coefficient.

For measuring large quantities of water flowing in pipes, the Venturi meter is at once the simplest and most accurate method available. With a meter that has been carefully rated, the probable error in computing the discharge should be less than 1 per cent.

In installing the meter for use on pipe lines where it is desired to keep a continuous record of the rate of flow, arrangement is often made whereby the difference in pressure heads, or even the quantity flowing, may be registered by a stencil on a cylindrical record actuated by clock mechanism.

87. The Pitot Tube. — This device, invented by Pitot in 1732, is essentially an instrument for measuring the velocity at a fixed point in a moving stream. In its simplest form it consists of a small tube having its lower end bent through 90°, so that it may be placed in the stream with its open end turned directly against the current, as shown in Fig. 95 *a*. The dynamic pressure against the orifice causes a head h to be maintained above the free surface of the water. If the tube be turned so that the plane of the orifice is parallel to the direction of the stream lines, the pressure on the orifice is static only, and the level of the water in the tube is very closely that of the free stream surface (Fig. 95 *b*). The theoretic relation between h

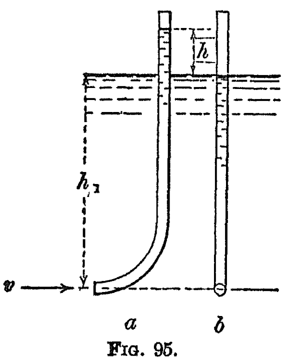

FIG. 95.

and v for the first position is that ordinarily stated between head and velocity,

$$h = \frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

Although an analytical proof for this is not possible, it seems logical to believe that, inasmuch as a head h will produce a velocity v, a stream with velocity v will maintain a head h by reason of the dynamic pressure it can furnish. Ample experimental proof of the above relation is furnished by the experiments of W. M. White, published in the *Journal of the Association of Engineering Societies*, August, 1901. Mr. White caused a vertical jet, from an orifice located in the horizontal

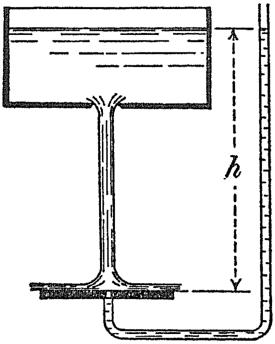

FIG. 96.

floor of a reservoir, to fall upon a flat circular plate placed at right angles to the jet (Fig. 96). Through the plate were drilled numerous holes, each one being connected with an open piezometer placed vertically. One of the latter is shown in the figure, connected with the hole that is under the central portion of the jet. In this piezometer the water stood at practically the same elevation as in the tank, showing that the dynamic pressure produced on the plate at the point where the piezometer was attached was just sufficient to sustain a column of water to a height equal to that through which the water in the jet had fallen. Since this height was the velocity head in the jet at the point where it met the plate, the truth of the above-stated relation seems apparent.

The first Pitot tubes were used in open channels and their form did not differ much from that shown in Fig. 95. Darcy improved the tube by making it conically convergent and adding a second tube having its point directed downstream (Fig. 97). In this the water stood at a lower level than the surface of the stream, and the difference h between the heads in the two tubes was used to determine v. For this form of tube it was necessary to write

$$h = c\frac{v^2}{2g}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

and to determine c by experimental rating of the tube. This was done by drawing the instrument through still water at known velocities and measuring the observed heads. Figure 97 shows the two tubes joined at their upper ends and fitted with an air-cock, a. This could be closed after the water in the tubes had found its levels, and the whole apparatus removed from the stream to more carefully measure h. A modification of Darcy's tube had the second point turned until its orifice lay in a plane parallel to the stream's motion. Theoretically the relation, $v = \sqrt{2gh}$ should have held good for this tube, but it was found that a coefficient was necessary as in equation (95). The coefficient varied widely with the construction of the tubes, and it was thought at the time that the form of the velocity point was largely responsible for this.

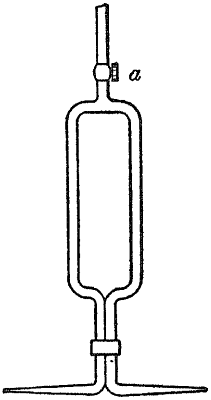

Fig. 97.

Although Darcy and Bazin have shown that, with proper care, the Pitot tube may be made to give excellent results when used in open channels, it has been but little used for that purpose. In 1888 Freeman used it to measure the velocities in the different parts of the cross-section of jets from fire nozzles, and there, too, it gave results which were highly satisfactory. It is very interesting to note that the orifice of his tube was formed from the point of a stylographic pen, having an opening of about 0.006 of an inch in diameter.

Recent experiments in this country have shown that Pitot's tube is well adapted to determining the velocity at any point in a closed pipe. Since, in this case, the velocity column has its height affected by the pressure in the pipe, it is necessary that a second tube be used to measure the pressure head. From the observed head in the other tube may be subtracted the pressure head to obtain that due to velocity alone.

Williams, Hubbell, and Fenkell used Pitot tubes, designed on this principle, in their well-known experimental work on the water mains of Detroit, in 1898, and developed a very ingenious and practical form of tube which may be inserted in the side of a pipe by means of a stuffing box tapped into it. Through the stuffing box the tube is free to move, thus allowing its orifice to be set at any desired point on the pipe's diameter. In order to more accurately measure the difference in head in the two tubes, they connected the water columns with a differential oil gauge of the type described in Article 14.

Mr. Cole, in his Pitometer, has used the Pitot tube as a continuous meter by causing the difference in head, as indicated by the differential gauge, to be recorded continuously by photographic means.

In 1901–2 experiments were conducted by Gregory, Maltby, and White with a view to determining what effect various forms of velocity and pressure openings have upon the reduction coefficient in formula (95), and seeing if it were not possible to design a form of tube that should have a constant coefficient of unity. Without relating in detail the various experiments, references to which appear at the end of this chapter, it may be said that Mr. White proved conclusively that the form of the impact point has little or no effect on the coefficient of the tube, and that velocity head is changed into pressure head according to the law $v = \sqrt{2gh}$, *provided* the impact surface surrounding the point is a surface of revolution having its axis parallel to the direction of the stream's motion. Messrs. Gregory and Maltby succeeded in designing a tube which, judged from their experiments, seems to have the desired constant coefficient of unity. At its lower end it consists of a long, slender tube drawn to a point of the proper size to admit

a very small tube, which is brazed into the larger one, forming the impact or velocity opening (Fig. 98). The small tube runs through the larger to its top and there connects with a pressure gauge, preferably of the oil differential type. The outer tube has three small openings in its side, spaced about

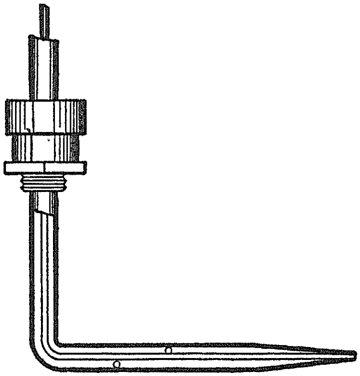

FIG. 98.

equally around the circumference, forming the openings for the pressure head. The top of the outer tube is also connected to the differential gauge.

Of the many forms experimented with, this tube alone seemed to possess the desired properties, and it was clearly shown that the usual variance in coefficient was due to incorrect design of the pressure openings rather than to the shape of the impact point. To obtain a correct pressure column it is necessary that no suction due to eddies or deflected water occurs at the pressure opening, since this would tend to decrease the pressure head and cause an observed velocity head *greater* than that existing in the moving stream. The fact that the coefficient used in equation (95) has for most tubes been found to have a value greater than unity, thereby leading to the incorrect assumption that the true relation between velocity and head should be $v^2 \div g = h$, may be thus explained.

When properly used the Pitot tube is a most valuable and accurate device for measuring velocities in closed channels, and in the hands of an expert should give results only exceeded in

accuracy by those obtained from the weighing tank. It should not be left unsaid, however, that any form of tube should, for reliable work, be carefully calibrated before use. To do this, it may be moved at known velocities through still water or placed in a pipe or stream whose velocity of flow is known. Williams used both methods in his Detroit experiments and found a difference in the coefficient thereby obtained. This would indicate that it is essential to rate a tube under conditions as nearly like as possible the conditions under which it is to be used. It is possible that the form of Williams' tube may have had something to do with the difference obtained, but further experiment is necessary to settle this point conclusively.

88. Variation in Velocity at a Cross-section. — So far in our work we have assumed that the velocity of the particles passing a section is the same at all points in the section. We have seen that this is merely an assumption, and that, on account of friction at the sides, the velocity decreases markedly from a maximum at the center to a minimum at the sides. Much experimental work has been done to ascertain the law which governs this variance, but, so far, it has not been definitely established. The usual method of procedure has been to use Pitot tubes (Art. 87), and measure the velocity at different points along a diameter. Under normal conditions of steady flow in a straight pipe, these velocities, if plotted from a reference line, *ab* (Fig. 99), should lie on a smooth curve, such as *cde*. From a study of such a plot it would then be possible to determine the nature and equation of the curve. The practical difficulties encountered are the irregularities found in the curves, due to errors in observation, and to pulsations in the flow, and the impossibility of making a measurement at the extreme edge of the pipe.

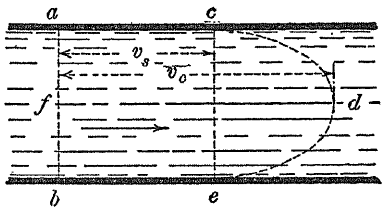

FIG. 99.

Darcy believed, as a result of his work on pipes ranging in

diameter from 7.8 to 19 inches, that the curve typical of the general case was a parabola with vertex at the pipe's axis. Assuming this to be so, we may deduce a relation between the center velocity v_c and the mean velocity v_m for the entire section.

From Fig. 99 it may be seen that the amount of water passing the section each second is represented by the solid generated by rotating the figure *acdeb* about *fd* as an axis. Its volume may be divided into two parts,—one a cylinder *aceb*, and the other a paraboloid *cde*. The volume of the cylinder is $\pi r^2 v_s$ and of the paraboloid, $\frac{1}{2}\ \pi r^2\ (v_c - v_s)$, since the volume of a paraboloid is one half that of the circumscribed cylinder. If we divide the total volume of the solid by the area of the pipe, we obtain

$$v_m = \frac{\pi r^2 v_s + \pi r^2 \left(\frac{v_c - v_s}{2}\right)}{\pi r^2} = \frac{v_s + v_c}{2}.$$

Experiments indicate that the value of the surface velocity is about one half that at the center, so that we may write

$$v_m = 0.75\, v_c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

If it were true that the parabola defines the curve of velocities, the placing of a Pitot tube at the center of a pipe would permit v_m to be at once calculated by this formula.

Recent experiments would indicate that the curve of velocities is more nearly an ellipse to which the sides of the pipe are tangent or nearly so. This is the conclusion of Bazin and also of Williams, Hubbell, and Fenkell. Bazin worked with a cement pipe 2.73 feet in diameter; and the latter three made their measurements on the cast-iron water mains of Detroit whose diameters were 12, 16, 30, and 42 inches. On the basis that the ellipse best represents the curve of velocities, and remembering that the volume of an ellipsoid is two thirds that of the circumscribed cylinder, we may obtain, by the method followed previously for the parabola, a value for v_m of

$$v_m = \frac{\pi r^2 v_s + 2\,\pi r^2 \dfrac{(v_c - v_s)}{3}}{\pi r^2} = \frac{v_s + 2\,v_c}{3}.$$

Assuming as before that $v_c = 2\,v_s$, we obtain

$$v_m = 0.83\,v_c. \qquad (97)$$

The Detroit experiments indicated the relation $v_m = 0.84\,v_c$, and this value was found to hold good for all the sizes experimented upon. Other experiments on pipes of smaller size seem to show a slight decrease in the ratio with decrease in diameter, but this is not certain.

89. Pipe Line with Nozzle. — The combination of a nozzle and pipe line is very common in engineering practice, the effect of the nozzle being to deliver the water at a higher velocity than would otherwise be obtained. Figure 100 illustrates the general

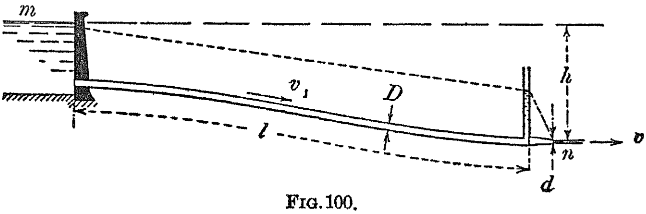

FIG. 100.

case, the water being supplied to the pipe from a reservoir. If we apply Bernoulli's Theorem to the points m and n, we obtain

$$h = \frac{v^2}{2g} + 0.5\frac{v_1^2}{2g} + f\frac{l}{D}\frac{v_1^2}{2g} + \left(\frac{1}{c^2} - 1\right)\frac{v^2}{2g}, \qquad (98)$$

the last term representing the head passing through the nozzle (Art. 54). In this equation appear two unknowns, v and v_1, but a relation between them being given by

$$\frac{v}{v_1} = \frac{D^2}{d^2}, \qquad (99)$$

a solution for either of them is possible.

Inasmuch as the velocity in the pipe is less than in the issuing jet, the pressure at the point of entrance to the nozzle must be greater than that due to atmosphere. Evidently the velocity of the jet will increase with the pressure at the base of the nozzle, and this latter will be a maximum when the velocity in

the pipe (and consequently the head lost by friction) is a minimum. This requires that the diameter of the pipe be as large as possible compared with that of the nozzle opening. In practice there are found problems which demand that the velocity of the jet be a maximum. More often, however, as in the case of a power wheel, it is necessary that the *amount of energy* furnished by the stream be a maximum. Since the energy is wholly in kinetic form, its value, for a stream having a cross-sectional area a and moving with a velocity v, is

$$\text{K. E.} = \frac{Mv^2}{2} = \frac{wav^3}{2g}.$$

If the nozzle area be increased, the amount of water discharged will be increased also, but the velocity of the jet will be diminished. The kinetic energy being a function of the quantity and velocity, it would appear that a maximum amount of energy will be obtained from the stream for some definite ratio of nozzle to pipe area. This ratio may be found as follows: If we neglect the slight loss at entrance and that occurring in the nozzle, the velocity of the jet as found from (98) and (99) will be

$$v = \left[\frac{2gh}{1+\frac{fld^4}{D^5}}\right]^{\frac{1}{2}}.$$

$$\therefore \text{K. E.} = \frac{wav^3}{2g} = \frac{62.5\,\pi d^2}{8g}\left[\frac{2gh}{1+\frac{fld^4}{D^5}}\right]^{\frac{3}{2}},$$

which may be written

$$\text{K. E.} = \frac{Kd^2}{\left[1+\frac{fld^4}{D^5}\right]^{\frac{3}{2}}} = Kd^2\left[1+\frac{fld^4}{D^5}\right]^{-\frac{3}{2}}.$$

To find the value of d which shall make K. E. a maximum, we have only to put the first derivative of K. E., with respect to d, equal to zero and solve for d. Thus, —

$$\frac{d\,(\mathrm{K.E.})}{dd} = 2Kd\left[1+\frac{fld^4}{D^5}\right]^{-\frac{3}{2}} - \frac{3}{2}Kd^2\left[1+\frac{fld^4}{D^5}\right]^{-\frac{5}{2}} \times \frac{4fld^3}{D^5} = 0,$$

from which

$$d = \sqrt[4]{\frac{D^5}{2fl}}. \qquad (100)$$

In using this equation a tentative value of f must be assumed and d found approximately. With this value of d the velocity may be computed and a closer approximation to f obtained. This operation may then be repeated if necessary.

It is worth noting that, under the conditions implied in equation (100), it can be easily proved that the head lost in pipe friction will be

$$\text{Lost head} = \frac{h}{3}.$$

The student should make no mistake in the interpretation of this statement or of equation (100). Given a *specified* diameter of pipe, the kinetic energy in the jet will be a maximum under the stated conditions. A *larger* amount of energy, however, may be developed from the same nozzle by using a larger pipe than specified. In practical power development the limiting size of pipe is reached when further increase in diameter causes an additional outlay, the interest of which will exceed the returns from the sale of the increase in power.

It is well for the student to note that in any case where an economical use of the total available head is desired, the pipe line should be free from abrupt changes either in section or direction.

90. Branching Pipes. — (1) *The Problem of Three Reservoirs.* Figure 101 shows a problem which sometimes arises in the design of water supply systems. A high level reservoir is to supply two others at lower levels by means of a main and branches. Given the length and diameter of each pipe and the levels of the junction and the reservoirs, it is desired to find the rate of discharge into the reservoirs.

It was pointed out in Art. 82, equation (81), that in the case of long pipes the loss by friction is so nearly the entire head h

on the discharging end, as to permit the law of flow to be represented by

$$h = f \frac{l}{d} \frac{v^2}{2g},$$

without incurring an error larger than is apt to result from the selection of a proper value for f.

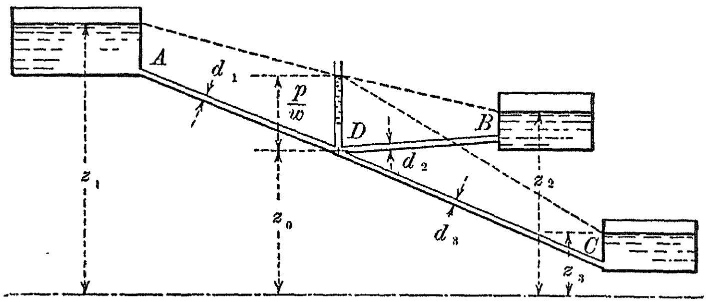

FIG. 101.

We may therefore apply this equation to the flow in the pipes *AD*, *DB*, and *DC*, obtaining respectively,

$$z_1 - \left(z_0 + \frac{p}{w}\right) = f \frac{l_1}{d_1} \frac{v_1^2}{2g}, \quad . \quad . \quad (1)$$

$$\left(z_0 + \frac{p}{w}\right) - z_2 = f \frac{l_2}{d_2} \frac{v_2^2}{2g}, \quad . \quad . \quad (2)$$

$$\left(z_0 + \frac{p}{w}\right) - z_3 = f \frac{l_3}{d_3} \frac{v_3^2}{2g}. \quad . \quad . \quad (3)$$

Since there are four unknowns appearing in these equations, $\frac{p}{w}$, v_1, v_2, and v_3, a fourth equation must be had for solution and this is furnished from the *equation of continuity*, —

$$a_1 v_1 = a_2 v_2 + a_3 v_3. \quad . \quad . \quad . \quad (4)$$

The solution involves the use of tentative values for f, which may be more closely found from the resulting values of v_1, v_2, and v_3.

The addition of (1) and (2) results in

$$z_1 - z_2 = f\frac{l_1}{d_1}\frac{v_1^2}{2g} + f\frac{l_2}{d_2}\frac{v_2^2}{2g}, \quad . \quad . \quad (5)$$

Similarly the addition of (1) and (3) gives

$$z_1 - z_3 = f\frac{l_1}{d_1}\frac{v_1^2}{2g} + f\frac{l_3}{d_3}\frac{v_3^2}{2g}, \quad . \quad . \quad (6)$$

which with (4) and (5) enables the flow in each pipe to be found.

It is to be noted that the above solution depends on the assumption that the flow is from reservoir A to both B and C. A little thought will enable the student to see that the dimensions of the pipes and the levels of the reservoirs might be so arranged that the intermediate reservoir at B would discharge through DB instead of being filled by it. If the pipe DB is to *deliver* water to the reservoir B, its gradient must slope in that direction, and this requires that the pressure head at the junction be such that $\left(z_0 + \frac{p}{w}\right) > z_2$. If $\left(z_0 + \frac{p}{w}\right)$ just equals z_2, then the water in DB will have no movement. If $\left(z_0 + \frac{p}{w}\right) < z_2$ the flow must be from B to D. In any problem, therefore, where the level of B is higher than the junction, it is necessary to first determine the direction of flow in BD. This may be done by assuming that $\left(z_0 + \frac{p}{w}\right)$ just equals z_2, so that in this pipe no flow occurs. We may then compute, for the resulting value of $\frac{p}{w}$, what the quantities Q_1 and Q_3 would be [equation (1) and (3)]. If Q_3 is found larger than Q_1, it means that flow must be from B to D. For, assuming the reverse to be true, the grade line for DB would slope as shown in Fig. 101, under which condition the actual value of $\frac{p}{w}$ would be greater than assumed, and Q_3 would be increased while Q_1 would be diminished. With Q_3 larger than Q_1 by the *first* assumption, and the difference in amount increased by the *second*, it can be seen that the second is not possible, since it implies $Q_1 = Q_2 + Q_3$.

Similarly, if on the first assumption Q_1 is found larger than Q_3, it means that the flow is from D to B. Having thus determined the direction of flow in DB, the discharge from DB and DC may be obtained by the use of the equations previously given. A method which seems preferable to this, however, because of its shortness, is as follows: The direction of flow having been determined, a value for $\frac{p}{w}$ may be *assumed* that will give a slope to the grade line of DB consistent with the found direction. Equations (1), (2), and (3) then serve to give Q_1, Q_2, and Q_3, and Q_1 should equal $Q_2 + Q_3$ or $Q_1 + Q_2 = Q_3$ according as the flow is into or from B. Assuming that Q_1 should equal $Q_2 + Q_3$, but is found to be greater than $Q_2 + Q_3$, then the value of $\frac{p}{w}$ has been assumed too small and must be modified to give the desired relation. Although seemingly a laborious method, it will be found that two or three trials will give a satisfactory solution and the time required will be less than that required to solve equations (4), (5), and (6) simultaneously.

This problem may be extended to any number of reservoirs and solved as above.

(2) *Problem of the Siamesed Pipes.* This problem is illustrated in Fig. 102, which shows a high-level reservoir connected

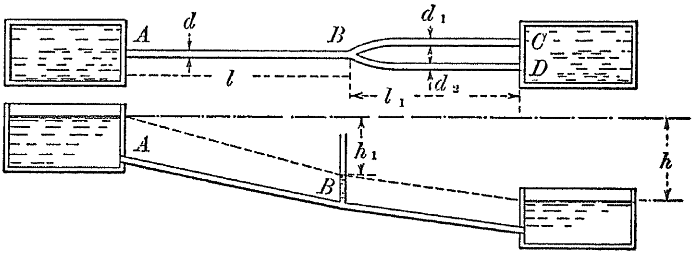

Fig. 102.

with a lower one by means of a pipe line whose lower portion consists of two pipes, of diameters d_1 and d_2, laid side by side, and entering the reservoir at the same level. Given the diameters and lengths of the pipes, also the difference in reservoir level, it is desired to find the rate of flow in the different pipes

of the system. Assuming that all the head is lost in friction, we have, for the pipe AB,

$$h_1 = f\frac{l}{d}\frac{v^2}{2g}, \quad \ldots \ldots \quad (1)$$

and for BC

$$h - h_1 = f\frac{l_1}{d_1}\frac{v_1^2}{2g}, \quad \ldots \ldots \quad (2)$$

while for BD

$$h - h_1 = f\frac{l_1}{d_2}\frac{v_2^2}{2g}. \quad \ldots \ldots \quad (3)$$

The addition of (1) and (2) results in

$$h = f\frac{l}{d}\frac{v^2}{2g} + f\frac{l_1}{d_1}\frac{v_1^2}{2g},$$

and (1) and (3) give

$$h = f\frac{l}{d}\frac{v^2}{2g} + f\frac{l_1}{d_2}\frac{v_2^2}{2g}.$$

These two equations contain three unknown velocities, but a third equation is furnished by the relation

$$av = a_1v_1 + a_2v_2.$$

The solution is then possible.

It should be noted that the head lost by friction in BC is the same as in BD [see (2) and (3)]. If the total loss in head between reservoirs be desired, it will be the loss in AB plus the loss in *one* of the branch pipes. In case it is difficult for the student to see that this is so, let him bear in mind that while in each of the branch pipes a certain head is lost, this loss is experienced *only by a portion* of the total quantity flowing between reservoirs.

If the branch pipes have the same diameter, then only two velocities are unknown and two equations will suffice. These are

$$h = f\frac{l}{d}\frac{v^2}{2g} + f\frac{l_1}{d_1}\frac{v_1^2}{2g},$$

and

$$av = 2a_1v_1.$$

(3) *Problem of Divided Flow.* — A common arrangement of piping is that shown in Fig. 103, where, for purpose of distribution, a main line is tapped by a smaller pipe which later returns to the main. Assuming that no water is drawn from the main or branch between B and E and that the draught at

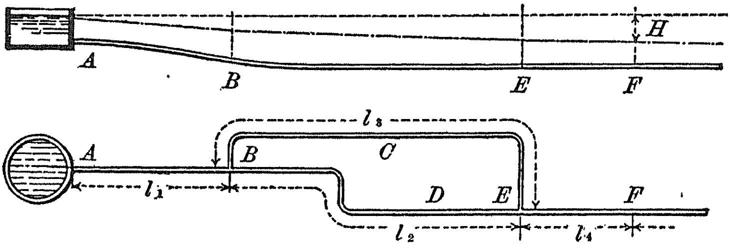

FIG. 103.

F, and beyond, causes the grade line at that point to lie at a distance H below the reservoir level, we may proceed to find the velocities of flow in the different pipes as follows. Since the terms *head* and *energy per pound of water flowing* are synonymous (see Art. 37), we may write as an expression for the total energy lost between A and F,

$$WH = Wf_1\frac{l_1}{d_1}\frac{v_1^2}{2g} + W_2f_2\frac{l_2}{d_2}\frac{v_2^2}{2g} + W_3f_3\frac{l_3}{d_3}\frac{v_3^2}{2g} + Wf_4\frac{l_4}{d_4}\frac{v_4^2}{2g},$$

in which W is the weight of water flowing per second in the system, and W_2 and W_3 the respective weights flowing in BDE and BCE. Next it should be noted that the heads lost in BDE and BCE are equal, each pipe having a common pressure-head at B and E, and consequently the same fall in their hydraulic grade lines. This gives

$$f_2\frac{l_2}{d_2}\frac{v_2^2}{2g} = f_3\frac{l_3}{d_3}\frac{v_3^2}{2g},$$

and if for $f_3\frac{l_3}{d_3}\frac{v_3^2}{2g}$ we substitute from this relation in the previous equation and divide the result by W (remembering that $W = W_2 + W_3$), we obtain

$$H = f_1\frac{l_1}{d_1}\frac{v_1^2}{2g} + f_2\frac{l_2}{d_2}\frac{v_2^2}{2g} + f_4\frac{l_4}{d_4}\frac{v_4^2}{2g} \quad . \quad . \quad . \quad (a)$$

This equation for lost head might at once have been written had it been clear to the student that the total head lost between B and E was that lost in *one* of the pipes and *not the sum* of the heads lost in both pipes. To obtain v_1, v_2, and v_4 from this equation, it is necessary to have other equations, and these are furnished by the relations

$$a_1 v_1 = a_4 v_4, \quad \text{(b)}$$

$$a_1 v_1 = a_2 v_2 + a_3 v_3, \quad \text{(c)}$$

and

$$f_2 \frac{l_2}{d^2} \frac{v_2^2}{2g} = f_3 \frac{l_3}{d_3} \frac{v_3^2}{2g} \quad \text{(d)}$$

The solution of these four equations simultaneously gives the desired velocities. Using the relations given in (*b*), (*c*), and (*d*), equation (*a*) may be written

$$H = \left[c_1 + c_2 c_3 \left(\frac{a_1}{a_2\sqrt{c_3} + a_3\sqrt{c_2}}\right)^2 + c_4\left(\frac{a_1}{a_4}\right)^2\right]\frac{v_1^2}{2g},$$

in which c_1, c_2, c_3, and c_4 have replaced the more cumbersome ratios,

$$f_1 \frac{l_1}{d_1}, \quad f_2 \frac{l_2}{d_2}, \text{ etc.}$$

In the first solution of this equation it is necessary that an average value for the different friction factors be assumed and afterward corrected according to the resulting velocities.

(4) *Main Line with Laterals.* — Whenever a pipe line becomes the main of a distributing system, the laterals of which lead from the main at irregular intervals along its length, a very complicated problem arises in computing the necessary size for the main so that the pressure will not fall at any point below a stated amount. A solution anything more than approximate is impossible unless all the data concerning the sizes of the laterals and the amounts of their maximum simultaneous withdrawals are known. As these are generally unknown, the following rude approximation may be made. Figure 104 shows such a main line supplied with water from a reservoir A. In the upper portion AB of the main there are no laterals, and the assumption will be made that from the point B to the end of the pipe at C the laterals are spaced equidistant on the main

and draw equal quantities of water from it. At C the main is dead-ended. The assumption of equal draught by the laterals causes a uniform reduction in velocity in the main

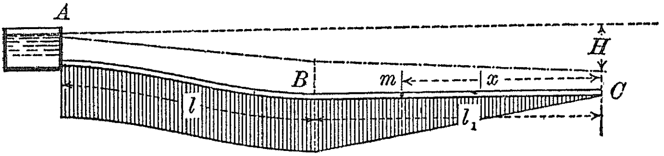

FIG. 104.

between B and C, the velocity at C being zero. In the portion AB the velocity is v and is constant. Below the pipe is sketched a shaded area to represent the variation in velocity in the main. It will be seen that at any point m, distant x feet from C, the velocity is

$$v_m = v\,\frac{x}{l_1}.$$

The lost head occurring in a dx length of pipe at this point would be

$$\text{L.H.} = f\,\frac{dx}{d}\,\frac{v^2}{l_1^2}\,\frac{x^2}{2\,g}.$$

To find the total head lost between B and C we may integrate the above expression with x a variable whose limits are zero and l_1. There results,

$$\text{Head lost in } BC = \frac{1}{3}\left(f\,\frac{l_1}{d}\,\frac{v^2}{2g}\right),$$

or the amount lost is one third that lost in an equal length of main containing no laterals. For the head lost in the entire main we have

$$\text{L.H.} = f\,\frac{l}{d}\,\frac{v^2}{2\,g} + f\,\frac{l_1}{3\,d}\,\frac{v^2}{2\,g}.$$

It will be noted that f is not constant for the total portion BC and whatever value is assumed for it will therefore be an approximation. In actual design, however, this need cause no serious error, since liberal allowance has to be made in choosing a value for f to provide for deterioration of the pipe. The lost head as given by the last equation may be subtracted from

the static head on the end of the pipe to give the pressure head at that point. Since the general requirement is to find the diameter of pipe necessary for fixed values of pressure head at C and quantity Q, we may substitute for v its equal, $\frac{4\,Q}{\pi d^2}$, obtaining for d

$$d = \sqrt[5]{\left(l + \frac{l_1}{3}\right)\frac{16 f Q^2}{2\,gH\pi^2}}.$$

As stated above, results obtained by the process of reasoning just given will be approximate only and should be modified to meet the peculiar conditions of each problem.

PROBLEMS

NOTE.—In the solution of the following problems, the value of f, if not given, should be determined on the basis of clean cast-iron pipe.

1. Determine the amount of fall to be given a 16-in. pipe line, 2000 ft. in length, in order that its discharge may be 10 cu. ft. per second.

✓ 2. Two reservoirs, with a difference in level of 90 ft., are connected by 2 miles of 15-in. pipe. Calculate the head lost in friction and the amount discharged.

✓ 3. A pipe line consists of two sections, one 500 ft. long and 12 in. in diameter, the other 1200 ft. long and 18 in. in diameter. If the change in section is abrupt and the amount discharged be 3 cu. ft. per second, find the loss in head in each section due to friction and the loss resulting from sudden enlargement of the pipe. Plot the hydraulic grade line.

✓ 4. Compute the diameter of pipe necessary for discharging 1500 gal. of water per minute, the pipe being 1000 ft. long and its discharging end 4 ft. lower than the surface of the reservoir supplying it.

5. The pressure head at one point in a 12-in. pipe line is 50 ft. At a point 1000 ft. beyond, in the direction of flow, the pressure is 20 lb. per square inch. If the discharge is 5 cu. ft. per second, and 2 ft. of head is lost at intervening bends, what is the slope of the pipe?

✓ 6. What head would be required for an 8-in. pipe line, 3000 ft. long, leading from a reservoir and terminating in a 2-in. nozzle, the required discharge being that corresponding to a velocity of flow of 6 ft. per second in the pipe? Assume velocity coefficient for nozzle at 0.95.

✓ 7. At a distance of 2000 ft. from the supplying reservoir, a 12-in. pipe is 140 ft. lower than the reservoir surface. How much water can be allowed to pass through the pipe (the entire length of which is much greater than 2000 ft.) without reducing the pressure at the point in question below 50 lb. per square inch? Assume $f = 0.02$.

8. A level pipe line is abruptly enlarged from 4 to 8 in. in diameter. The velocity in the 4-in. pipe being 16 ft. per second, how much energy is wasted in heat at the enlargement? If the pressure head in the 4-in. pipe is 50 ft., what will it be in the 8-in.?

9. An 18-in. pipe is discharging 3000 gal. per minute. At a point 1000 ft. from the supplying reservoir the center of the pipe is 80 ft. below the reservoir surface. What pressure, in pounds per square inch, is to be expected there during flow?

10. Water flows through a 24-in. pipe 5000 yd. in length. At 1000 yd. it yields up 300 cu. ft. per minute to a branch. At 2800 yd. it yields up 400 cu. ft. per minute to a second branch. At 4000 yd. it yields up 600 cu. ft. per minute to a third branch. The delivery at the end is 500 cu. ft. per minute. Find the head absorbed by friction.

11. Reservoir A is at elevation 1000 ft. Thence an 8-in. pipe line leads 3000 ft. to elevation 800, at which point it branches into two lines: a 6-in. line running 2000 ft. to reservoir B, elevation 850, and a 6-in. line running 1000 ft. to reservoir C, elevation 875. At what rate will water be delivered to each reservoir? Assume $f = 0.02$.

12. A 12-in. pipe, 8000 ft. long, is connected with a reservoir whose surface is 250 ft. above the pipe's discharging end. If for the last 4000 ft. a second pipe of the same diameter be laid beside the first and connected to it, what would be the increase in discharge? Assume $f = 0.02$.

13. Assuming the data of the preceding problem, find the increase or decrease in discharge resulting from inserting in the original pipe line a section of 18-in. pipe 2000 ft. long. (Note that no restriction is made regarding the location of the enlarged portion.) Loss by sudden change in section may be neglected, and f assumed equal to 0.02.

14. A pipe 30,000 ft. long and 6 ft. in diameter supplies 10 nozzles with water from a reservoir whose level is 507 ft. above the nozzles. Each nozzle has an opening of 6 sq. in. and a coefficient of discharge and velocity of 0.95. Assuming f to be 0.017, find the aggregate horse power available in the jets.

15. How many horse powers are being transmitted through a 3-in. pipe in which the velocity of flow is 15 ft. per second and the accompanying gauge pressure 40 lb. per square inch?

16. A 2-in. nozzle is attached to a 6-in. pipe line. Pressure at the base of nozzle during flow is 60 lb. per square inch. The coefficient of discharge of the nozzle is 0.90. With these data compute the horse power of the jet.

17. At what rate in horse powers is energy being transmitted past a cross-section in a pipe line 24 in. in diameter, where the mean velocity is 10 ft. per second and the gauge pressure 50 lb. per square inch? Supposing the pipe to terminate just beyond the gauge in a nozzle whose coefficient of velocity is 0.97, what will be the corresponding velocity of the jet?

18. Find the maximum power to be transmitted by water in a 36-in. pipe, the metal being $1\frac{1}{2}$ in. thick, the allowable stress 2800 lb. per square inch, and the velocity of flow 1 ft. per second. If the pipe is $1\frac{1}{4}$ miles in length, find the loss of power.

19. The tip of a Pitot tube being placed at the center of a 24-in. pipe, the difference in height of the velocity and pressure columns is found to be 22.3 in. On the assumption that the curve of velocities is an ellipse, and that the surface velocity is one half that at the center, compute the discharge from the pipe. Coefficient of tube, 0.91.

20. A fire engine with a 12-in. pump cylinder supplies water to a nozzle through 500 ft. of 3-in. hose. What horse power at the pump will be necessary to maintain a stream of water having a velocity of 75 ft. per second with the nozzle held 30 ft. above the pump cylinder? The nozzle has a diameter of $1\frac{1}{2}$ in. at the tip and a coefficient of 0.90. The value of f for the hose may be assumed at 0.017.

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CHAPTER IX

FLOW IN OPEN CHANNELS

91. Divisions of the Subject. — The term "open channel" is here used to include not only natural streams and all artificial canals, but also all forms of closed conduits, aqueducts, and pipes which flow only *partly full.* A distinct difference between such channels and a pipe flowing full is that the latter depends upon an external head for the production of flow, while the former depends solely upon the slope given to it.

The analytical treatment of open channel flow is more difficult and unsatisfactory than the flow in pipes because of the wide range in the conditions which present themselves. A pipe is generally circular in form and the roughness of its lining is the chief variable. With an open channel not only is there found to be a wide variance in the nature of the lining, but also the form of cross-section may be of an almost infinite variety of shapes and may change from point to point even in the same channel. Under these circumstances it can be readily seen that it is exceedingly difficult to derive a formula for flow that shall be perfectly general in its application. Indeed it has so far been impossible to do so, and there is little or no likelihood of such a formula being discovered.

Broadly viewed, all problems of open channel flow may be classified under the following divisions: —

1. Flow in Artificial Channels.
   (*a*) Uniform Flow.
   (*b*) Non-uniform Flow.
2. Flow in Natural Channels.

92. Artificial Channels, Uniform Flow. — This is the case with which the engineer has mostly to do. The assumption of

steady flow is made so that the quantity of water passing any transverse section of the stream is constant. To make the flow *uniform*, the mean velocity of flow past such a section must be the same for all positions of the section. This necessitates that the cross-sectional area of the stream be constant throughout its length, so that along any longitudinal section the water must have a constant depth. The surface is therefore parallel to the bed, both having some angle of inclination α with the horizontal. We shall speak of this inclination as the *slope* of the channel and express it as

$$s = \frac{h}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

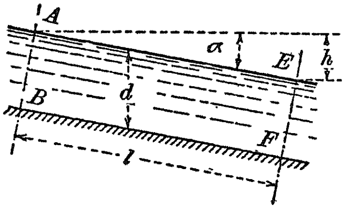

FIG. 105.

h being the vertical fall in the length of channel l, and s the sine of α (Fig. 105). In any cross-section $ABCD$ (Fig. 106), that part of the channel lining which comes in contact with the stream is known as the *wetted perimeter* and has the designation p. The hydraulic radius or hydraulic mean depth is therefore (Art. 85)

$$r = \frac{A}{p}.$$

With the above assumptions made, we may regard the water between any two sections, A-B and E-F (Fig. 105) as a solid prism having a uniform motion down the inclined trough of the stream. The forces producing or hindering motion are its weight Awl, the end pressures P_1 and P_2, and the frictional resistance R offered by the sides of the trough (Fig. 107). The forces P_1 and P_2 must be equal, since they represent the pressures on two equal areas under the same pressure head. Since they are opposite in direction, we need not consider them. The component of the prism's weight

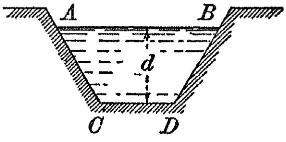

FIG. 106.

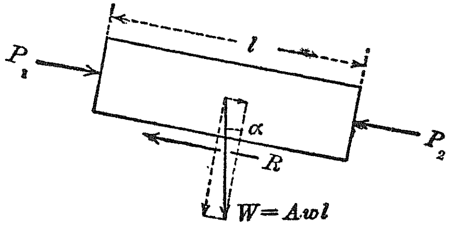

FIG. 107.

W, along the line of motion, being $Awl \cdot \sin\alpha$ and the motion being *uniform*, we have

$$Awl \cdot \sin\alpha - R = 0.$$

If F represents the value of the frictional resistance per unit area of rubbing surface, so that $R = F \times pl$, the above may be written

$$F = \frac{Awl \cdot \sin\alpha}{pl} \text{ or } F = wrs. \quad . \quad . \quad . \quad (102)$$

Our knowledge of fluid friction, as outlined in the previous chapter (Art. 76), shows F to *approximately* equal some function c of v^2. If then we replace F by cv^2 and solve for v, we obtain

$$v = C\sqrt{rs}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

in which C has replaced $\sqrt{\frac{w}{c}}$. This formula is known as Chezy's formula, being identical with the one deduced in Art. 85 to express the law of flow in pipes. At first glance the two formulæ would seem to differ as, in the case of the pipe, s represents the slope of the hydraulic grade line, while in the open channel it represents the slope of the bed or of the stream's surface. For the open channel the grade line lies in the stream's surface, since the latter would mark the heights of all open piezometer columns which might be placed in it. The two formulæ are, therefore, identical.

In the derivation just given all fairly rational treatment of the problem ceased with the assumption that $F = cv^2$. This relation we have seen to be only an approximate one. If it were strictly true, C in Chezy's formula would be constant for any particular channel, varying only with the roughness of its

lining. Experiments show that this is not the case, and that C varies also with the slope and the hydraulic radius; and that it may be made approximately a constant by giving to r and s fractional exponents which are more closely in agreement with facts than is the exponent one half. Thus has it been proposed to write

$$v = C r^x s^y,$$

and considerable work has been done by experimenters toward determining proper values for x and y. The chief difficulty arising in the use of the formula lies in the fact that every channel differs more or less from any other, and consequently has values of its own for x and y. It is necessary, therefore, that we have at our command complete tables showing the varying of x and y with the nature of the channel, and up to the present time experimental data are not sufficient for the purpose. So far the bulk of reliable experimental work has been directed toward determining the value C in the Chezy formula, and a brief outline of the more important of these investigations will now be given.

93. Determination of C by Ganguillet and Kutter. — In 1869 Messrs. Ganguillet and Kutter made a most comprehensive study of all the reliable open-channel experiments which had been made up to that time, and from them deduced a formula for C which is the only general rule of wide application that we have. The experiments from which they drew their conclusions ranged from observations on small artificial channels up to measurements made on the Mississippi River. Their formula is as follows: —

$$C = \frac{41.65 + \dfrac{0.00281}{s} + \dfrac{1.811}{n}}{1 + \left(41.65 + \dfrac{0.00281}{s}\right)\dfrac{n}{\sqrt{r}}}. \quad . \quad . \quad . \quad (104)$$

It is seen that the value of C is made to vary with the slope, the hydraulic radius, and a quantity n which may be termed the *coefficient of roughness*. The latter depends upon the channel lining and has values approximately as follows: —

Manning's Formula :— $C = \frac{1.486}{n} r^{1/6}$. or $V = \frac{1.486}{n} \times r^{2/3} s^{1/2}$.

Channel Lining	n
Planed timber, glazed or enameled surfaces	0.009
Smooth clean cement	.010
Unplaned timber, new brickwork well laid	.012
Smooth stonework, iron, and ordinary brickwork	.013
Rough ashlar and good rubble masonry	.017
Firm gravel	.020
Earth in ordinary condition	.025
Earth with stones, weeds, etc.	.030
Earth or gravel in bad condition, strewn with detritus	.035

Kutter's formula has been widely used by American, German, and English engineers and may be relied upon to give excellent results when applied to normal channels. By a normal channel is meant one whose slope, hydraulic radius, or velocity of flow has no unusual or extreme value. A great deal has been claimed for the formula, since it was designed to apply to not only regular channels of uniform section, but also to natural rivers. Inasmuch as the basis of the formula is the assumption of steady, *uniform flow*, it would hardly seem probable that it can be depended upon to give accurate results if applied to a channel whose general shape, slope, or hydraulic radius is constantly changing. The most that can be said for it when so used is that it may serve to roughly approximate the flow, and give a result which is liable to be much in error. It is quite safe to say that any formula which is derived for steady, uniform flow in regular, artificial channels, is hardly suitable for use with large, irregularly shaped, natural streams. Kutter's formula is without doubt the best yet devised when consideration is taken of the fact that it may be applied to almost any kind of surface and be relied upon to give good results. The student is warned, however, against placing too much reliance upon results obtained from any empirical formula, and in this particular case errors as large as 5 per cent are to be expected, and 10 per cent errors are apt to occur. Values of *C* based on this formula are given in Table X of the Appendix.

94. Bazin's Determination of *C*.—In 1897, H. Bazin (*Annales des Ponts et Chaussées*, 1897) made a most elaborate and careful discussion of his own and all other reliable experiments and

When r is less than 3.8, Kutter's Coefficient is always greater than Manning.

n of Kutter's formula (which is the same as the mannings) is not the same as Bazin's m.

proposed a formula for determining the value of C which may, for English units of measure, be written as follows: —

The same

$$C = \frac{87}{0.552 + \frac{m}{\sqrt{r}}} \quad \ldots\ldots \quad (105)$$

$\frac{7}{1.811 \frac{m}{\sqrt{r}}}$ (gama) (Hyd. radius)

The quantity r is the hydraulic radius and m is a coefficient of roughness like the n in Kutter's formula. Values for m may be taken as follows: —

Channel Lining	m
For very smooth cement surfaces or planed boards	0.06
For unplaned boards, well laid brick, or concrete	.16
For ashlar, good rubble masonry, or poor brickwork	.46
For earth beds in perfect condition	.85
For earth beds in fair or ordinary condition	1.30
For earth beds in bad condition, being covered with sticks, stones, weeds, and other detritus	1.75

Values of C based on Bazin's formula are given in Table XI of the Appendix. It should be noted that Bazin's formula makes C independent of the slope s, the author believing that the latter had little influence on the coefficient. The formula is much more simple than Kutter's, and is favored by French and some English engineers. If it be compared with Kutter's by applying them both to the same channel, it is found that they agree fairly well for ordinary channels having no unusual dimensions and falling within the limits of Bazin's observations. Bazin's formula finds preferable application in the case of small channels. To urge, as has been done, that his formula is better to use because of its mathematical simplicity, is no good reason for its adoption to the exclusion of others. Kutter's formula has been graphically solved by Professor I. P. Church in a book of neat diagrams,* and where constant use is made of the formula, much saving of time and labor may be accomplished by their use.

95. Variation in the Coefficient *C*. — Having seen that C is a variable dependent upon the slope, hydraulic radius, and largely the lining of the channel, it will be of interest to note the com-

* Wiley & Sons, Publishers, New York.

ad in pipes due to friction $h = \frac{f l v^2}{d\,2g} = c\sqrt{rs}$

mon limits to the values which it will ordinarily be found to have. These may be approximately defined as follows: —

For very smooth cement or timber	C = 125 to 150
For ordinary timber, concrete, brickwork, etc. . . .	100 to 130
For rough masonry or firm gravel	70 to 90
For earth bed in good condition	50 to 75
For earth beds in bad order	20 to 50

Of course these limits should be considered as very flexible, and for any particular channel, C should be figured by careful use of one of the above formulæ.

Example. — Compute the flow in a circular, brick-lined conduit 5 ft. in diameter having a slope of 1 in 1000. Conduit flows half full.

$$r = \frac{A}{p} = \frac{d}{4} = 1.25 \text{ ft.} \qquad s = 0.001.$$

By Kutter, $(n = 0.013)$,

$$C = \frac{41.65 + \dfrac{0.00281}{0.001} + \dfrac{1.811}{0.013}}{1 + \left(41.65 + \dfrac{0.00281}{0.001}\right)\dfrac{0.013}{\sqrt{1.25}}} = 121.$$

$$v = 121\sqrt{1.25 \times .001} = 4.2 \text{ ft. per second.}$$

$$Q = 9.82 \times 4.2 = 41.2 \text{ cu. ft. per second.}$$

By Bazin, $(m = 0.16)$,

$$C = \frac{87}{0.552 + \dfrac{0.16}{\sqrt{1.25}}} = 125.$$

$$v = 125\sqrt{1.25 \times 0.001} = 4.4 \text{ ft. per second.}$$

$$Q = 9.82 \times 4.4 = 43.2 \text{ cu. ft. per second.}$$

96. Most Advantageous Cross-section.—If an open channel has its slope s and cross-sectional area A *fixed*, it is evident that the maximum velocity (hence maximum discharge) will occur when the area A is so proportioned that the surface of contact between water and channel lining will be as small as possible

(frictional resistance being reduced to a minimum). This is the same as saying that p must be a minimum. It may be further seen that this condition will also permit using the *least possible slope* consistent with the demand that a fixed Q be discharged. Therefore, in general, a very efficient section is obtained by making p a minimum. Remembering that $r = \frac{A}{p}$, the making of r a maximum obtains the same result.

Since of all figures having equal areas the circle has the least perimeter, it follows that a semicircular section has the smallest value of p (Fig. 108 a). Such a section is difficult to build

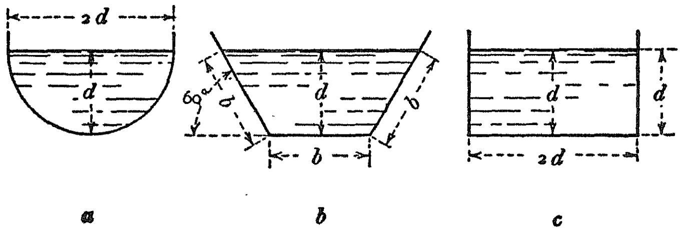

FIG. 108.

and to maintain under some conditions, and generally the trapezoidal or rectangular form is used. Since of all rectangles the square has the least perimeter for a fixed area, it will be advantageous to make the rectangular section a half square (Fig. 108 c). Similarly, if it is to be trapezoidal, p will be least for a half hexagon (Fig. 108 b). For each of the three sections the hydraulic radius is one half the water depth. In unlined channels it is necessary to use the trapezoidal section if the earth banks are to maintain their form. The angle α (Fig. 109) having been determined consistent with the nature of the soil, it will be found that the best proportions between the base width b and the depth d will occur when a semicircle, having its center in the water surface, may be inscribed tangent to the three sides of the section. Again the hydraulic radius is one half the water depth. Furthermore, for any fixed

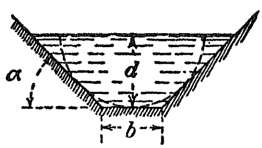

FIG. 109.

depth d, the hydraulic radius will remain constant, as α is made to vary, if the sides are kept tangent to the circle.

For any form of section, the best proportions may be found by expressing r in terms of the fixed area and one of its dimensions. The putting of the first differential of this expression equal to zero will permit of the solution of the equation for the value of this dimension. This may be illustrated as follows:

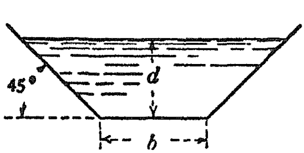

FIG. 110.

We will suppose it is desired to find the relation between d and b (for "best section") for the channel shown in Fig. 110, maintaining a fixed area of 100 sq. ft. We have

$$(a)\quad A = (b + d)d = 100.$$

$$(b)\quad r = \frac{100}{b + 2d\sqrt{2}}.$$

$$(c)\quad b = \frac{100 - d^2}{d}.$$

Combining (c) and (b), we have

$$r = \frac{100d}{100 - d^2(1 - 2\sqrt{2})},$$

and placing $\frac{dr}{dd} = 0$, there results $d = 7.4$ ft.

From (c) we then obtain $b = 6.1$ ft.

97. Non-uniform Flow. — We have seen that uniform flow necessitates constant slope and cross-section in the channel. The mean velocity past all cross-sections, and the depth of water are also constant.

Now it may happen that, owing to the presence of peculiar conditions, all these characteristics of uniform flow will be found lacking. The flow will be *steady*, giving for all sections the relation

$$Q = a_1v_1 = a_2v_2 = a_3v_3 = \text{etc.},$$

but the mean velocity will be found to vary from section to section, and the form and area of the cross-sections will change.

To illustrate, let Fig. 111 represent in section a channel constructed to connect two large ponds whose surfaces stand at different levels. The bed of the channel is *horizontal* and we will assume the pond levels to remain constant. Flow will take place in some such manner as indicated, the depth at A-B

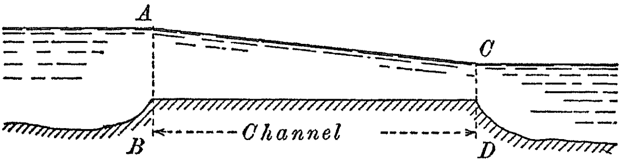

Fig. 111.

being in excess of that at C-D. The cross-section and the velocity of the stream is, therefore, constantly changing between these points. The flow is *steady* but non-uniform, and it cannot be made uniform until, by adjusting the slope to the flow, the bed of the stream and the water surface are *parallel*. In the figure given, the bed was chosen horizontal to better illustrate the problem; but from the above it may be seen that it may slope and yet not have the right slope to carry the water with uniform motion. Figure 112 shows a portion of the channel under such conditions. If v_1 and v_2 represent the velocities at the points A and B, and z_1 and z_2 are the heights of the points above the datum, we may write (Bernoulli's Theorem)

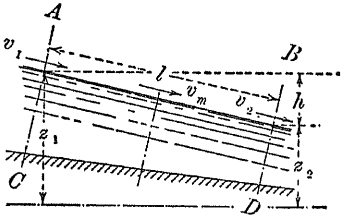

Fig. 112.

$$\frac{v_1^2}{2g} + z_1 = \frac{v_2^2}{2g} + z_2 + \text{Lost head},$$

and since

$$z_1 - z_2 = h,$$

we have

$$\text{Lost head} = h + \frac{v_1^2}{2g} - \frac{v_2^2}{2g}. \quad . \quad . \quad . \quad . \quad (106)$$

To otherwise express the lost head let us return for the

moment to the case of uniform flow. By simple transposition of the Chezy formula (Art. 92), we obtain

$$s = \frac{v^2}{C^2 r},$$

or, since

$$s = \frac{h}{l},$$

$$h = \frac{lv^2}{C^2 r}. \qquad (107)$$

Here h is the drop of the water surface or hydraulic grade line in any length of stream l. It therefore is the measure of the lost head in this distance. (The student should review Art. 92 in this connection and verify this statement.)

Returning now to the problem in non-uniform flow, it is seen that in the length l we have a constantly changing value for v and r; but we will assume that the head lost in this case can be expressed in the same form as (107), provided we give v and r values that apply to a section taken midway between the two end sections, A-C and B-D. This value of the lost head being substituted in (106), there results, after transposition,

$$\frac{v_1^2}{2g} + h = \frac{v_2^2}{2g} + \frac{lv_m^2}{C^2 r_m}, \qquad (108)$$

and this is a *general* equation for non-uniform flow.

If the area of the middle section be taken as the mean of A_1 and A_2, so that

$$A_m = \frac{(A_1 + A_2)}{2},$$

then it can be easily shown that

$$v_m = \frac{2 v_1 v_2}{v_1 + v_2}.$$

For the value of r_m, if we assume the wetted perimeter p_m to be equal to $\frac{1}{2}(p_1 + p_2)$, which we may do without sensible error, we have

$$r_m = \frac{A_1 + A_2}{p_1 + p_2}.$$

Also we may write

$$v_1 = \frac{Q}{A_1} \text{ and } v_2 = \frac{Q}{A_2}.$$

The substitution of the above values in equation (108) will give

$$Q = \frac{\sqrt{2gh}}{\sqrt{\left(\frac{1}{A_2}\right)^2 - \left(\frac{1}{A_1}\right)^2 + \frac{8gl}{C^2} \cdot \frac{(p_1 + p_2)}{(A_1 + A_2)^3}}}. \tag{109}$$

By measuring the areas and perimeters of two cross-sections, a distance l apart, and also the drop h in this distance, we may by formula (109) compute approximately the quantity flowing in the channel.

The value of C must be found from formula (104) or (105), using r_m as the hydraulic radius and $h \div l$ for the slope.

98. Non-uniform Flow. Change in Depth. — A problem which sometimes arises in non-uniform flow is the determination of the change in water depth which takes place from section to section. Assuming a state of flow as shown in Fig. 113, let d_1 and d_2 be the depths at A and B respectively. The drop in the channel bed between sections at A and B is $l \sin \alpha$, l being the distance between sections, and α the angle which the bed makes with the horizontal. We may then express the drop h in the stream's surface as

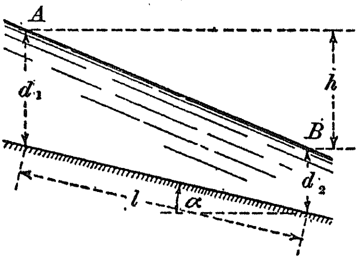

FIG. 113.

$$h = d_1 + l \sin \alpha - d_2,$$

and this value substituted in equation (109) permits that equation to be written

$$l = \frac{(d_1 - d_2) - \left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]\frac{Q^2}{2g}}{\left[\frac{p_1 + p_2}{(A_1 + A_2)^3}\right]\frac{4Q^2}{C^2} - \sin \alpha}. \tag{110}$$

This equation permits us to find the distance l between two ross-sections whose depths differ by $(d_1 - d_2)$. It is necessary hat the slope of the bed, the form and dimensions of the two ections, and the discharge Q be known.

99. Backwater. — A common problem in non-uniform flow, ecessitating the application of the foregoing principles, occurs vhen a channel is obstructed by a dam or weir. If the water vere flowing with uniform motion previous to the construction f the weir, the raising of the latter will cause the water to set ack up the stream, increasing the depth, and the flow in that art of the channel will become non-uniform. It will differ rom the case previously cited, in that the depth will *increase* n the direction of flow. Figure 114 shows the conditions, ab

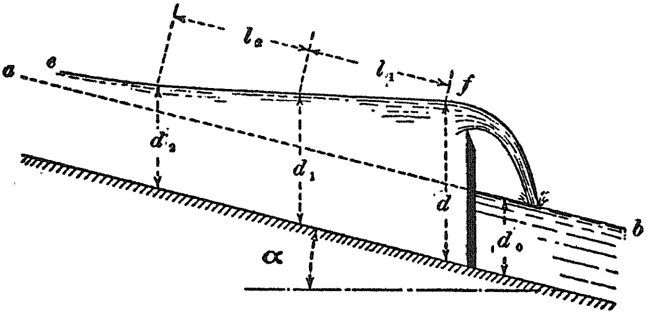

FIG. 114.

eing the original surface of the stream (parallel to the bed), nd ef the new surface or *curve of backwater*. Two important uestions arise : —

1. How much will the water be raised at a given distance pstream from the point of obstruction ?

2. How far upstream will the influence of backwater be elt ?

For non-uniform flow with *increasing* depth, we may proceed s in Art. 98, when deriving equation (110), and obtain for and l_2 (see Fig. 114),

$$l_1 = \frac{(d - d_1) - \left[\frac{1}{A_1^2} - \frac{1}{A^2}\right]\frac{Q^2}{2g}}{\sin\alpha - \left[\frac{p + p_1}{(A + A_1)^3}\right]\frac{4\,Q^2}{C^2}}, \tag{111}$$

and

$$l_2 = \frac{(d_1 - d_2) - \left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]\frac{Q^2}{2g}}{\sin\alpha - \left[\frac{p_1 + p_2}{(A_1 + A_2)^3}\right]\frac{4\,Q^2}{C^2}}. \tag{112}$$

Similarly, equations may be written for l_3, l_4, etc., giving the distances in which successive decrements in depth $(d_2 - d_3)$, $(d_3 - d_4)$, etc., occur. By starting with the known depth just back of the weir, and assuming a series of decrements in depth $(d - d_1)$, $(d_1 - d_2)$, etc., we may use these equations for determining the distances l, l_1, l_2, etc., in which these decrements occur. The first of our two questions may in this way be approximately answered.

Example. — A rectangular channel 42 ft. wide contains water 3.5 ft. deep, flowing with a mean velocity of 4 ft. per second. By the construction of a weir, the water level is raised $1\frac{1}{2}$ ft. at the weir. Assuming $n = 0.02$ in Kutter's formula, find the distances upstream to points where the backwater is 1 ft. and .5 ft. deep respectively.

Since the slope $(\sin\alpha)$ is not given, it will be necessary to find it by a series of approximations. Assuming, therefore, that $C = 90$ in the Chezy formula, $v = C\sqrt{rs}$, we have $4 = 90\sqrt{3\,s}$, from which s is found to be 0.00066. If C *really does* equal 90 for this channel, then we should obtain such a result from Kutter's formula, using values of $s = 0.00066$, $n = 0.02$, and $r = 3$. Actually the formula gives $C = 88$ for these values. A second solution of Chezy's formula with $C = 88$ results in $s = 0.00069$. This value is only slightly in excess of that previously found, and a probable value will not be far from 0.0007. We will assume this value and use it for $\sin\alpha$ in equations (111) and (112). The value of C in these equations is not to be taken the same as found for the channel when *uniform* flow existed, but should be calculated for each stretch of

stream, l_1, l_2, etc., from the *mean* hydraulic radius of each section. By Art. 97,

$$r_m = \frac{A + A_1}{p + p_1},$$

giving $r_1 = 3.87$ and $r_2 = 3.53.$

The values of C corresponding are

$$C_1 = 93, \quad C_2 = 92.$$

Our data therefore are as follows: —

$d = 5$	$d_1 = 4.5$	$d_2 = 4$
$A = 210$	$A_1 = 189$	$A_2 = 168$
$p = 52$	$p_1 = 51$	$p_2 = 50$
	$r_1 = 3.87$	$r_2 = 3.53$
	$C_1 = 93$	$C_2 = 92$

$$Q = 42 \times 4 \times 3.5 = 588.$$

Substitution in equations (111) and (112) results in:

$$l_1 = \frac{0.50 - (0.0000280 - 0.0000227)\ 5370}{0.0007 - 0.000262} = 1080 \text{ ft.}$$

$$l_2 = \frac{0.50 - (0.0000354 - 0.0000280)\ 5370}{0.0007 - 0.000363} = 1370 \text{ ft.}$$

We therefore have: —

Depth of backwater at weir $= 1.5$ ft.
Depth of backwater 1080 ft. above weir $= 1.0$ ft.
Depth of backwater 2450 ft. above weir $= 0.5$ ft.

Other decrements in depth may be assumed and the corresponding distances figured, but the method is somewhat laborious. A simpler method for solving the problem will be given in the next paragraph.

100. Equation of Backwater Curve. — Several more simple formulæ than those above given have been devised, based on a treatment of the problem using calculus. If the method just used be applied to sections distant dx apart so that the differ-

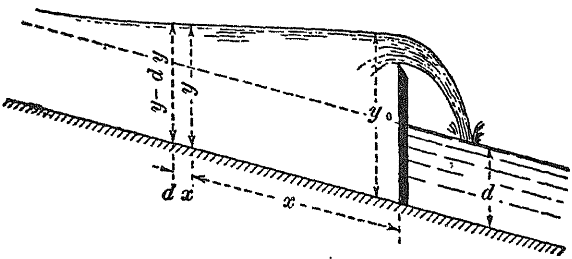

FIG. 115.

ence in depth at these sections is dy (Fig. 115), a differential equation may be formed which, when integrated, gives the following as the equation of the backwater curve: —

$$x = \frac{y_0 - y}{s} + d\left(\frac{1}{s} - \frac{C^2}{g}\right)(\phi - \phi_0). \quad . \quad . \quad (113)$$

ϕ and ϕ_0 are used to represent complicated functions of $\frac{y}{d}$ and $\frac{y_0}{d}$ respectively. A table of their values, based on a more extended one calculated by Bresse, is given below. The equation is in the form usually found in hydraulic literature, but a slightly simpler form may be obtained if for C^2 we substitute $\frac{v^2}{rs}$ (from Chezy's formula) and assume that r may be written equal to the depth d, an assumption which is practically a truth for *broad, shallow* streams. Proceeding thus, we obtain

$$x = \frac{1}{s}\left[y_0 - y + \left(d - \frac{v^2}{g}\right)(\phi - \phi_0)\right]. \quad . \quad . \quad (114)$$

TABLE OF BACKWATER FUNCTIONS

$y \div d$	ϕ	$y \div d$	ϕ	$y \div d$	ϕ	$y \div d$	ϕ	$y \div d$	ϕ
1.00	∞	1.30	.373	1.60	.218	1.90	.147	2.40	.0894
1.05	.896	1.35	.335	1.65	.203	1.95	.139	2.50	.0821
1.10	.680	1.40	.304	1.70	.189	2.00	.132	2.60	.0757
1.15	.561	1.45	.278	1.75	.177	2.10	.119	2.70	.0701
1.20	.480	1.50	.255	1.80	.166	2.20	.107	2.80	.0649
1.25	.420	1.55	.235	1.85	.156	2.30	.0978	2.90	.0604

Example. — It will be interesting to apply this equation for amplitude of backwater to the previous numerical problem and compare results with those obtained by the more approximate and cumbersome equations of Art. 99. The ratio $\frac{y_0}{d}$ is $\frac{5}{3.5} = 1.43$, and ϕ_0 corresponding is found from the table to be 0.288. At the point where the backwater has decreased in depth by 0.5 ft., the ratio of $\frac{y}{d}$ is $\frac{4.5}{3.5} = 1.285$, and by the table, ϕ is 0.387. The value of s is 0.0007 as before and $\frac{v^2}{g}$ is 0.497.

$$x = [0.50 + (3.5 - 0.497)(0.387 - 0.288)] \div 0.0007 = 1140 \text{ ft.}$$

Similarly for a point where the depth of backwater has decreased by 1.0 ft.,

$$x = [1.0 + (3.5 - 0.497)(0.581 - 0.288)] \div 0.0007 = 2690 \text{ ft.}$$

These results are quite close to those obtained by the more approximate method. If the distance upstream to the point where the limit of backwater is reached be desired, we have $\frac{y}{d} = 1.0$, and for this value the table gives ϕ as ∞. This shows that the curve of backwater is asymptotic to the original surface of the stream and theoretically has no end. In applying this more exact method to natural streams, it should be remembered that its derivation was based on the assumption of a regular rectangular channel having great width compared with its depth. Therefore the results should not be looked upon as more than roughly approximate, save in exceptional cases where the stream is quite regular.

101. Flow in Natural Channels. — A discussion of this division of open channel flow will not be given here. While much has been done in this important field of hydraulics, all efforts to originate an accurate, or reasonably accurate, formula for flow which will apply to streams of all kinds and sizes have so far come to naught. It is claimed that Kutter's formula may be applied to natural streams, but with the complicated and constantly changing conditions which a natural channel presents,

it is beyond reason to expect that anything more than mere approximations may be obtained by the application of formulæ. Dependence, for accurate results, must be placed on direct measurements, made in the field, from which the discharge may be computed. Two methods are in general use. A weir may be constructed and the discharge measured by aid of the principles given in Chapter VII, or measurements of the velocity may be made with floats or current meters. The weir method is undoubtedly the more accurate, but on streams of considerable size the construction of a proper weir is expensive if not quite impossible. In using floats or meters, the operation is based on the following principle: If the total cross-sectional area A of a stream be divided into partial areas A_1, A_2, A_3, A_4, etc., through which the current moves with velocities v_1, v_2, v_3, v_4, etc., respectively, then the total discharge may be written

$$Q = A_1v_1 + A_2v_2 + A_3v_3 + A_4v_4 + \text{etc.} \quad . \quad . \quad (115)$$

The division of a cross-section into areas of known size is easy, once that soundings on the section have been made and the form and size of the section determined. Measurements of the velocities through these areas is effected by means of a current meter, which is in simplest form a wheel with vanes that revolve with a speed proportional to that of the moving water. The relation existing between revolutions per second and current velocity is obtained by previously "rating" the meter.

In this operation the meter is moved through still water at different velocities and the revolutions per second noted. A rating table or curve may then be constructed showing the above relation.

When use is made of floats, a straight and regular section of the stream is sought, and two cross-sections chosen a known distance apart. Floats are put into the stream above the upper section, and the times of their transit from one section to the other are noted by a stop watch. The velocity thus measured is the velocity of the stream in that portion through which the float ran. A float commonly used is the subsurface float, so made that it may be made to run at any desired depth, and its position located by means of a small surface float attached to

it by a fine wire or cord. If the float be adjusted to run at about six tenths of the water depth, experiments have shown that it will move with approximately the mean velocity of all the water particles in the vertical strip through which it runs. The cross-section being divided into a series of such strips

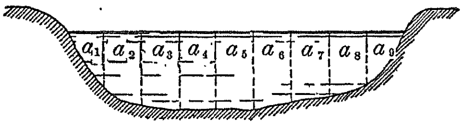

FIG. 116.

(Fig. 116), the calculation of the discharge may be effected by aid of equation (115).

If the cross-section of the stream be fairly constant for the reach chosen, rod floats may be used. These consist of hollow tubes, so weighted at the end as to float immersed their entire length. Provided each float reaches well down to the bottom of the stream, its velocity may be assumed equal to the mean of the strip in which it runs. In using floats it is necessary to know the mean depth along a longitudinal section of the run, and to that end soundings are taken on the end sections and others chosen intermediate.

The details of making a proper gauging are too numerous and lengthy to be given a place here, and at the end of this chapter will be found references to standard literature on the subject.

PROBLEMS

1. A canal, in firm clean earth, has a bottom width of 20 ft., and side slopes of 2 to 1. If the water depth be 4 ft., and the slope of the bed 1 in 1500, compute the probable velocity of flow and the discharge.

2. A rectangular channel, 18 ft. wide and 4 ft. deep, has a slope of 1 in 1000, and is lined with good rubble masonry ($n = .017$). It is desired to increase as much as possible the amount of flow without changing the slope or *form* of section. The dimensions of the section may be changed, but the new channel must contain the same amount of lining as the old. Find the dimensions of the new section and compute the probable increase in the flow.

3. A trapezoidal canal is to have a base width of 20 ft., and side slopes of 1 to 1. The nature of the soil (clean earth) limits the velocity of flow to

2 ft. per second. What slope must be given the bed in order to deliver 182 cu. ft. per second?

4. Solve problem (1), using both Bazin's and Kutter's coefficient.

5. A canal is to have a trapezoidal section with one side vertical and the other sloping at 45°. It is to carry 900 cu. ft. per second, with a mean velocity of 3 ft. per second. Determine the depth and the bottom width which would require the least slope of bed.

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CHAPTER X

DYNAMIC ACTION OF JETS AND STREAMS

102. Principle of Momentum. — In this chapter will be discussed the action of moving streams with regard to the forces which they exert upon bodies that constrain them in their motions. A few fundamental principles in mechanics are involved and will be briefly reviewed.

If a mass be acted upon by a constant force, it suffers an acceleration in the direction of the force which may be determined from the relation $P = Ma$. The acceleration being uniform,

$$a = \frac{\text{Change in velocity}}{\text{Time}} = \frac{\Delta v}{\Delta t},$$

and

$$P = M\frac{\Delta v}{\Delta t}.$$

If the change in velocity be that which occurs in one second, then

$$P = M \Delta v, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

and the change in Mv, or *change in momentum*, is equal to the value of the constant force which produced it, or the *average* value if the force is not constant.

103. Application to Fluid Masses under Conditions of Steady Flow. — As an aid to the use and understanding of the above principle when dealing with fluid masses, the following demonstration will be found helpful. Figure 117 shows a portion of a gradually enlarging channel containing water in steady motion. As a consequence of the gradual change in sectional area, the velocity of the contained water particles, and hence their momentum, is gradually reduced. The velocity at some section m we may assume to be v_1, while at n it has been reduced to v_2.

Suppose that in the time dt particles at m move to m' and those at n to n', so that the distance mm' equals v_1dt and nn' equals v_2dt. The aggregate momentum of all the particles between m and n suffers a diminution which may be represented in the difference found between the momentum of the mass mm' and nn'. That this is so may be seen if it is noted that the aggregate momentum of all particles between m' and n remain constant. (The student should here review the discussion on page 44 relative to the change in kinetic energy taking place in the mass A-B, Fig. 39.) The momentum of the mass mm' being $\left(\frac{wa_1v_1dt}{g}\right)v_1$,

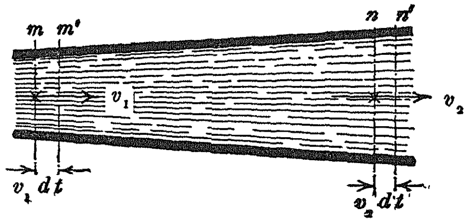

FIG. 117.

and that of nn' $\left(\frac{wa_2v_2dt}{g}\right)v_2$, we have from Art. 102,

$$P = \left(\frac{wa_1v_1^2dt}{g} - \frac{wa_2v_2^2dt}{g}\right) \div dt,$$

or, since $\qquad a_1v_1 = a_2v_2 = Q$, and $\frac{Qw}{g} = M,$

$$P = M(v_1 - v_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

The total change in momentum brought about between any two points in one second of time may therefore be expressed as the product of the mass passing a section in one second and the change in velocity between the two points. If a change in momentum in a direction parallel to a designated axis be desired, then the above formula will give that change *provided components of* v_1 *and* v_2, *parallel to the axis*, be used in place of v_1 and v_2. This principle will be freely used in the discussion of the following problems.

104. Force acting upon a Jet from an Orifice. — In the case of a jet issuing from an orifice, the velocity v, which all the particles are assumed to have, must be due to the action of a pres-

sure or force exerted by the water in the reservoir upon the particles at the base of the jet. The magnitude of the force may be found as follows. Figure 118 shows an orifice in the side of a reservoir from which the water issues with a velocity v. Conditions of flow having been made steady by providing a suitable inflow at e, equation (117) may be used to find the change in momentum, *in a horizontal direction*, that takes place in one second between sections at cd and mn, and the value of the force producing it. The value of the horizontal velocity component at cd being zero, and that at mn equal to v, we may write

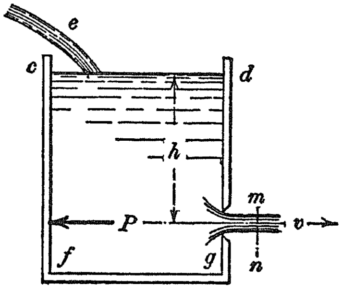

FIG. 118.

$$P = M(v - 0) = Mv. \quad . \quad . \quad . \quad . \quad (118)$$

The force P must be the resultant of all the horizontal forces acting on the mass $cdmn$ in the direction of motion, and as the pressure of the walls cf and dg on the contained water are the only such forces, P must be the difference in these, or the pressure at the base of the jet. Considering the opposites of these two forces, we see that the water presses on the wall cf with a force greater, by the amount P, than it does on dg, so that the reservoir is subjected to an unbalanced force which would cause it to move to the left with uniformly accelerated motion were it free to do so. This may be regarded as the *Reaction* of the jet.

Since $M = \frac{wav}{g}$, another form of expression for equation (118) would be

$$P = 2\,wa\frac{v^2}{2\,g} = 2\,awh. \quad . \quad . \quad . \quad . \quad (119)$$

From this it appears that the force is *twice the static pressure* that would exist on a plug just filling the orifice. This condition is a theoretic one, however, since in deriving (119) from (118) no allowance was made for the contraction of the jet or for the

effect of friction upon the velocity. For a standard orifice, assuming $c_v = 0.98$, $c_c = 0.62$, and $c_d = 0.61$, we have

$$W = 0.62\,wav,\ v = 0.98\sqrt{2\,gh},$$

$$\text{and } \frac{v^2}{2\,g} = (0.98)^2 h,$$

v being the actual velocity in the contracted section. Under these conditions

$$P = \frac{0.62\,wav^2}{g} = 2[0.62\,wa \times (0.98)^2 h] = 1.19\,awh.$$

Mr. Peter Ewart, of England, made an actual measurement of P, using a standard orifice, and obtained $P = 1.14\,awh$.

105. Energy of a Jet. — If a jet or stream move with a velocity v, we have already seen (Art. 37) that the kinetic energy per pound of water is $\frac{v^2}{2\,g}$. Accordingly if past any cross-section W pounds of water pass in one second, the total kinetic energy in the stream may be written

$$K = W\frac{v^2}{2\,g},$$

or, in the more usual form,

$$K = \frac{Mv^2}{2}. \qquad (120)$$

106. Force exerted by a Jet upon a Deflecting Surface. — If a jet be turned from its path by meeting tangentially a deflecting surface (Fig. 119), it exerts upon the surface a dynamic pressure P. We may consider the equal and opposite force P' to have been the cause of the deflection, and measure it as follows: Assuming the surface to be smooth, the jet will pass over it with practically undi-

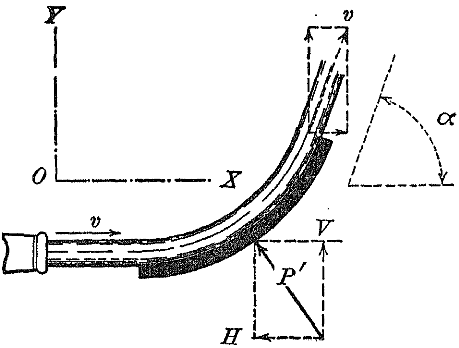

FIG. 119.

minished velocity. The H component of P' may be found by equating it to the total change in momentum effected in one second between the points where the jet enters upon and where it leaves the surface, this momentum being calculated in a direction *parallel* to the original direction of the jet. The velocity at the first named point being v, and the horizontal velocity component at the second point, $v \cos \alpha$, we may write

$$H = M(v - v \cos \alpha)$$

or

$$H = Mv(1 - \cos \alpha). \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

Similarly we may prove that the V component of P' has a value

$$V = Mv \sin \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

For the value of P' we have

$$P' = \sqrt{V^2 + H^2} = Mv\sqrt{2(1 - \cos \alpha)}. \quad . \quad . \quad . \quad (123)$$

107. Special Cases. — *Case I. Flat Plate Perpendicular to Jet* (Fig. 120). In this case the angle through which the water is deflected is 90°, and by the application of equation (121) we find that

$$P' = Mv. \quad . \quad . \quad . \quad (124)$$

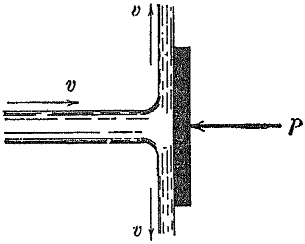

FIG. 120.

There is no resultant vertical pressure on the plate, and for this reason P' has the same value as its H component. The same result might be reached by noting that the stream's momentum in its initial direction is wholly destroyed and that the force which caused it must be given by the relation

$$P' = Mv.$$

Case II. Jet turned through 180°. — If the surface be so formed as to cause the jet to be completely turned back upon itself (Figs. 121 and 122), we have $\alpha = 180°$, and by equation (121)

$$P' = Mv\,[1 - (-1)] = 2\,Mv. \quad . \quad . \quad . \quad (125)$$

The force exerted by the jet upon the surface is thus twice as great as though directed normally against a flat plate. Applying the principle of momentum directly, we find that the total change in momentum, in one second, while passing over the

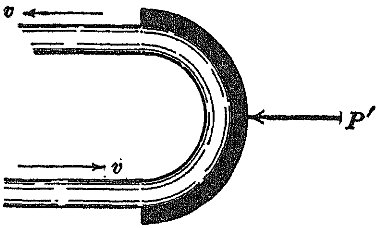

FIG. 121.

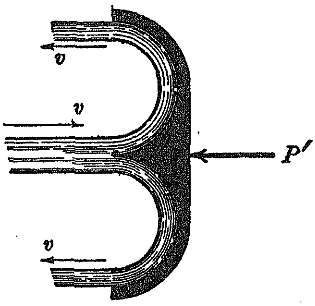

FIG. 122.

surface is $M[v - (-v)]$, or $2Mv$. The force necessary to produce this change is therefore given by equation (125).

108. Force exerted by a Jet upon a Moving Curved Surface. — (Fig. 123.) Here the curved surface shown in Fig. 118 is assumed to have a motion in the same direction as the jet but a lesser velocity, v_0. The motion is also uniform, a suitable resistance R preventing any acceleration. At the point A, where the jet is tangent (preventing impact) to the surface, the veloc-

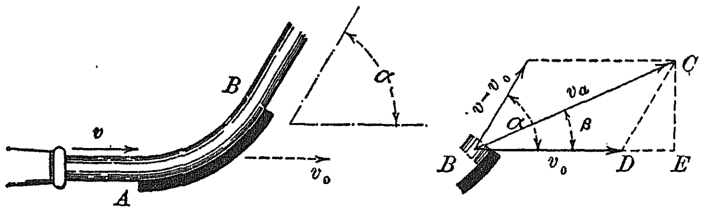

FIG. 123.

ity of the jet *relative to the surface* is $v - v_0$. If the surface be smooth and the vertical distance between A and B small, the relative velocity at B is also $v - v_0$. Therefore a particle arriving at B has not only a velocity of $v - v_0$ relative to the surface, but also a velocity of translation v_0. Its *absolute* velocity, v_a, is, therefore, represented in the second figure by the

diagonal of the parallelogram of velocities. The component of v_a in a direction parallel to the initial direction of the jet is $v_a \cos \beta$, and the total change in momentum in this direction, effected in one second while passing over the surface, is $M'(v - v_a \cos \beta)$.

The jet being horizontal, we have

$$\begin{aligned} H \text{ comp. of } P' &= M' (v - v_a \cos \beta) \\ &= M' \{v - [v_0 + (v - v_0) \cos \alpha]\} \\ &= M' (v - v_0)(1 - \cos \alpha). \quad . \quad . \quad . \quad (126) \end{aligned}$$

By a similar process of reasoning we find the value of the V component of P' to be

$$V \text{ comp. of } P' = M' (v - v_0) \sin \alpha. \quad . \quad . \quad (127)$$

Here M' is the mass of water which flows over the vane in one second of time and is *not* the same in value as M, which is the mass of water discharged each second by the jet. However, if we arrange a *series* of vanes, following one behind the other, so that, as each vane in succession cuts off the water from its predecessor, the water so cut off from the jet is left free to expend its kinetic energy on the first vane, then *all* the water in the jet may be utilized and M' be replaced by M in equations (126) and (127).

A common application of these principles is found in the construction and use of the so-called "Impulse wheels" for power development. Here the vanes or buckets are set at close intervals around the periphery of a wheel, and each receives in turn a pressure from a jet which is directed tangentially against the wheel. A section through one of the buckets, taken on a plane passing through the jet and perpendicular to a radius of the wheel, is sketched in Fig. 124. If the value of P is to be as large as possible for any given speed of the bucket, it will be a maximum when $\alpha = 180°$, in which case equation (126) becomes

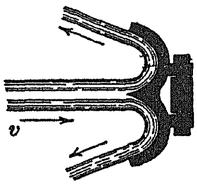

FIG 124.

$$P = 2 M(v - v_0). \quad . \quad . \quad . \quad . \quad . \quad . \quad (128)$$

Equation (126) is used to determine P since it is the H component of pressure that turns the wheel and performs work. The V component is practically radial and performs no work.

109. Power exerted on Moving Vanes.—In Fig. 123 the only motion taking place was in a horizontal direction, parallel to the jet. Therefore, for the work done upon the vane in one second, we have

$$L = H \text{ comp. of } P' \times v_0 = M'(v - v_0)(1 - \cos\alpha)v_0, \quad (129)$$

M' being equal to $wa(v - v_0) \div g$. If we make v_0 a variable in this expression and differentiate L, putting $\frac{dL}{dv_0} = 0$, we shall obtain as a value for v_0,

$$v_0 = \frac{v}{3},$$

which value will make L a maximum for any particular vane angle α.

If a *series* of vanes be used, M' assumes a value $M = \frac{wav}{g}$, (see Art. 108), and the first derivative of $\frac{dL}{dv_0}$ when put equal to zero gives a value for v_0 of

$$v_0 = \frac{v}{2}.$$

The formula for power then becomes

$$L = \frac{Mv^2}{4}(1 - \cos\alpha). \quad . \quad . \quad . \quad . \quad . \quad (130)$$

If the vane angle α be made 180°,

$$L = \frac{Mv^2}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (131)$$

which will be recognized as the expression for kinetic energy in the jet. That is to say, we are now utilizing all the water and all the kinetic energy which the jet has.

110. Dynamic Pressure in Pipe Bends.—If a pipe in which water flows with a velocity v be bent on a curve through some angle α, a resultant pressure will exist on the bend as in the

case of a jet deflected by a curved surface. The treatment of the two problems is identical, and for the resultant pressure we have (Art. 106),

$$P = Mv\sqrt{2(1-\cos\alpha)}.$$

The direction of the resultant will bisect the angle β. With a jointed pipe this pressure tends to disrupt the joints, and if the

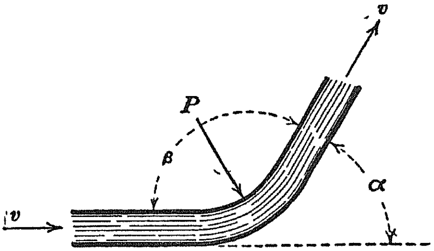

FIG. 125.

pipe be not embedded in firm material, it should be protected against rupture by clamps or bands.

Example. — A 36-inch pipe, discharging 50 cu. ft. per second, contains a one-piece bend of 90°. Find the stress in two anchor rods placed radially at the ends of the bend.

$$v = 50 \div \frac{\pi \times 9}{4} = 7.1 \text{ ft. per second.}$$

By equation (121)

$$\text{Stress in one rod} = \frac{50 \times 62.5 \times 7.1}{32.2}(1 - \cos 90°) = 690 \text{ lb.}$$

111. Stress in Pipe Produced by Change in Section. — If a pipe be changed in section, as shown in Fig. 126, the stream

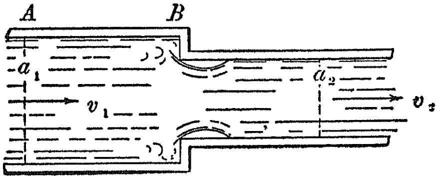

FIG. 126.

within exerts a force which tends to rupture it transversely. The magnitude of the force may be found as follows: Due to

the sudden change in section, the velocity is increased from v_1 to v_2, and a total change in momentum, equal to $M(v_2 - v_1)$, takes place in each second of time. This must be caused by an unbalanced force acting in the direction of motion. Three forces go to make up this resultant, the force P which the pipe exerts to retard the stream's motion, and the pressures, a_1p_1 and a_2p_2, existing in the pipe's cross-sections. Since the resultant force is measured by the change in momentum, we have

$$M(v_2 - v_1) = a_1p_1 - a_2p_2 - P,$$

or

$$\frac{wQ}{g}(v_2 - v_1) = Q\frac{p_1}{v_1} - Q\frac{p_2}{v_2} - P,$$

and finally

$$P = Q\left[\frac{p_1}{v_1} - \frac{p_2}{v_2} - w\left(\frac{v_2 - v_1}{g}\right)\right]. \quad . \quad . \quad . \qquad (132)$$

Example. — A pipe carrying 4 cu. ft. per second suffers a reduction in section from 2 sq. ft. to 1 sq. ft. If the pressure head in the full section be 30 ft., find the dynamic force exerted upon the pipe.

From the data,

$$v_1 = 2 \text{ ft. per second.}$$
$$v_2 = 4 \text{ ft. per second.}$$
$$p_1 = 62.5 \times 30 = 1875 \text{ lb. per square foot.}$$

Between points located axially in the two pipes we may write

$$\frac{v_1^2}{2g} + \frac{p_1}{w} = \frac{v_2^2}{2g} + \frac{p_2}{w},$$

neglecting any loss of head. From this, p_2 is found to be 1865 lb. per square foot.

$$\therefore P = 4\left(\frac{1875}{2} - \frac{1865}{4} - \frac{62.5 \times 2}{32.2}\right) = 1872 \text{ lb.}$$

112. Water Hammer in Pipes. — It has long been a well-known fact that the sudden stopping of the flow of water in a pipe will cause a rise in pressure in that portion of the pipe lying between the source of flow and the gate or valve by which the flow is checked. In some cases the rise in pressure may be

so large as to cause a bursting of the pipe, and it becomes a matter of importance that we determine the laws which govern the appearance of this phenomenon.

That we may better understand what actually takes place under such conditions, Fig. 127 has been drawn, showing a long

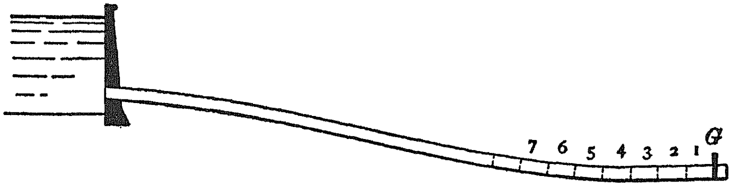

FIG. 127.

pipe line of constant diameter leading from a reservoir of water, and provided at its further end with a quick-closing valve G.

Under normal conditions the flow through the pipe is steady, having a velocity v past all sections. If the gate be made to close *instantaneously*, the particles of water in immediate proximity to it will have their velocity at once reduced from v to zero. If the whole mass of water in the pipe were inelastic (rigid) and contained in pipe walls that were inelastic also, then all the particles of water would likewise be instantaneously brought to rest and the pressure against the gate and all through the pipe would be infinite. This follows from the relation

$$\text{Force} = \text{Mass} \times \text{Acceleration},$$

the acceleration in this case being

$$a = \frac{dv}{dt} = \frac{v - 0}{0} = \text{Infinity}.$$

But the pressure is not infinite, due to the fact that the water is elastic, therefore compressible, and the pipe walls are elastic also. To understand the effect which these conditions have, let us assume the pipe to be divided into a great number of thin laminæ, some of which are shown, greatly exaggerated in size, in the above figure. When the gate closes, lamina 1 crowds up against it and is compressed by virtue of its own kinetic energy. As it is compressed, the ring of pipe wall surrounding it is distended by the increased pressure in the lamina. While the first lamina is being compressed and shortened,

lamina 2 follows on behind with undiminished velocity until the compression of lamina 1 is complete. It then suffers retardation and compression, at the same time stretching the pipe wall around it. Other laminæ follow in succession, so that in a very short time conditions are as shown in Fig. 128. The

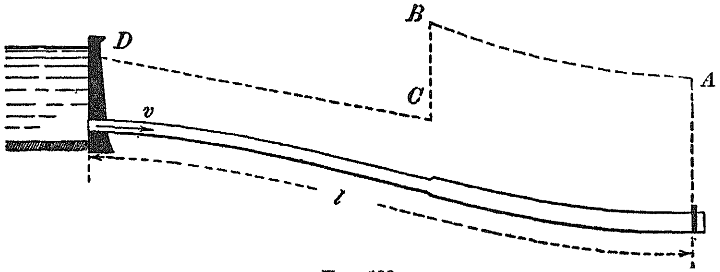

FIG. 128.

pressure in the distended portion is constant and the hydraulic grade line is some line *ABCD*. In the *unstretched* portion of the pipe the velocity v is *still maintained*. When the last lamina at the entrance to the pipe is compressed, the pipe is filled with water under a pressure much in excess of that which previously existed. If the length of the pipe be l, and t seconds have elapsed between the time of closure of the valve and the compression of the last lamina, then a wave of pressure has swept up the pipe with a velocity $v_p = l \div t$. The numerical value of this velocity varies with the density of the water and the thickness and elasticity of the pipe walls.

With the last lamina brought to rest, the total kinetic energy, which the water had, has been transformed and stored up in the elastic deformation of the water and pipe-walls. To more easily describe the continuance of the phenomena let us say that the attainment of this condition marks the end of the first period.

Second Period. This condition of inequipotential cannot be maintained, for as soon as the last lamina is compressed it will expand and the ring of pipe around it will contract, thus forcing the lamina out of the pipe into the reservoir, where it will resume its original condition of form and pressure. This follows with each lamina in succession until number 1 is reached

and the whole pipe has been restored to its original size and the water in it to its original normal pressure. Each lamina will also have acquired its original velocity v, but the motion will be in the *opposite direction*, toward the reservoir.

Third Period. The kinetic energy of the water column, now moving toward the reservoir, will be expended in lowering the pressure in the laminæ (commencing with number 1 and continuing until the last lamina is reached) to a point *below normal*, and the water in the pipe will again come to rest.

Fourth Period. Water from the reservoir now enters the pipe. The nearest lamina resumes normal pressure and moves with its original velocity toward the gate. The other laminæ follow in succession, and finally all are moving with velocity v in the original direction and under normal pressure. Conditions are now the same as at the beginning of operations. We have, therefore, completed a cycle of four movements, each one extending through the whole length of pipe. Two successive movements may be termed a "round trip" of the pressure wave. Another cycle follows the first and so on until the pressure wave dies out by reason of friction.

We shall now proceed to determine the value of p, the pressure in excess of that which existed at any point in the pipe during flow.

In Fig. 129, $ABCD$ represents a prism of water caught in the end of the pipe by the instantaneous closing of the gate G.

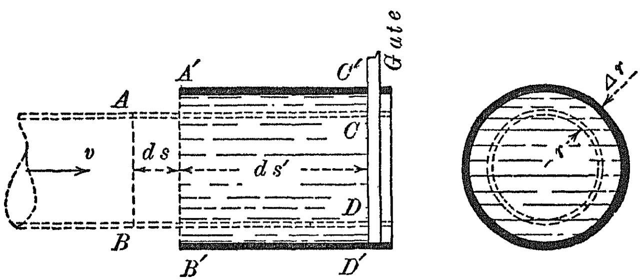

FIG. 129.

It contains many of the laminæ mentioned above, and by their successive compression the prism is shortened by the amount ds. Its final length becomes ds', and by reason of the stretch-

ing of the pipe, its radius is increased to $(r + \Delta r)$. For the change in volume we may write

$$\Delta V = A \cdot ds - 2\,\pi r \cdot \Delta r \cdot ds',$$

where A is the cross-sectional area of the pipe before distention, and $2\,\pi r \times \Delta r$ represents approximately the annular area added by the distention. The modulus of elasticity E is therefore (Art. 2),

$$E = \frac{pA(ds + ds')}{A \cdot ds - 2\,\pi r \cdot \Delta r \cdot ds'}. \quad . \quad . \quad . \quad (133)$$

Here p represents the intensity of the excess pressure arising from the compression of the laminæ. The stress produced in the pipe wall by this pressure is (Art. 21) $p' = pr \div t$. If under this stress the length of the circumference is stretched a distance Δl, we have for E', the elastic modulus of the pipe wall material,

$$E' = \frac{p'}{\Delta l \div 2\,\pi r} = \frac{pr \cdot 2\,\pi r}{\Delta l \cdot t}. \quad . \quad . \quad . \quad (134)$$

Since a circumference increases with the radius, we may write

$$\Delta l : 2\,\pi r = \Delta r : r,$$

or

$$\Delta l = 2\,\pi \Delta r.$$

Equation (134) may then be written

$$\Delta r = \frac{pr^2}{tE'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

Let us now return to a consideration of the prism $ABCD$. Each of its laminæ is brought to rest from a velocity v by a pressure on its forward face varying in intensity from zero to p. If, now, we imagine the first lamina to have reached its maximum pressure, then we may think of the entire mass $ABCD$ behind it as being brought to rest by the force pA acting on its end face. In coming to rest the center of gravity of the mass will have moved through a distance $ds \div 2$. Now the space traversed by a body while being brought to rest from an initial velocity v under the action of a constant force is

$$s = v^2 \div 2 \times \text{Acceleration}.$$

The acceleration (negative) of the mass $ABCD$ being

$$a=\frac{\text{Force}}{\text{Mass}}=\frac{p\cdot A\cdot g}{A(ds+ds')w},$$

we may substitute in the above formula, obtaining

$$\frac{ds}{2}=\frac{1}{2}\cdot\frac{v^2A(ds+ds')w}{pAg},$$

or more simply, since $\qquad \dfrac{ds}{dt}=v,$

$$p\cdot g\cdot dt=(ds+ds')wv. \quad . \quad . \quad . \quad . \quad (136)$$

The quantity ds being very small compared with ds', we may write without appreciable error,

$$p=\frac{wv}{g}\times\frac{ds'}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (137)$$

But $ds' \div dt$ represents the velocity with which the pressure wave moves along the pipe, and its value may be determined as follows: If we substitute in (133) the value of Δr as given in (135) and for A its value πr^2, we may combine (133) and (136), eliminating $(ds+ds')$. There results,

$$\frac{ds}{dt}-\frac{2\,rp}{tE'}\times\frac{ds'}{dt}=\frac{p^2g}{Evw}. \quad . \quad . \quad . \quad . \quad (138)$$

But $ds \div dt$ we know to be v, and for the value of p we may substitute from (137).

$$\therefore \text{Vel. of pressure wave}=\frac{ds'}{dt}=\sqrt{\frac{g}{w}\cdot\frac{EE't}{tE'+2\,rE}}. \quad (139)$$

This value of $ds' \div dt$ substituted in (137) gives finally for the excess pressure,

$$p=v\sqrt{\frac{wEE't}{g\,(tE'+2rE)}}. \quad . \quad . \quad . \quad . \quad (140)$$

It is of interest to note that this formula gives p as independent of the *length of pipe* and consequently of the *mass of water flowing in it.* It is to be remembered, however, that the formula is based on the assumption that the closure of the gate is *instantaneous.*

In 1897–98 Professor Joukovsky of Moscow, Russia, carried out an elaborate and most carefully conducted series of experi-

ments on water hammer, using pipes of 2, 4, 6, and 24 inches diameter. Their lengths were 2494, 1050, 1066, and 7007 ft. respectively. His experimental results substantiated the truth of the above derived formulæ and showed the effect which slow closure of the gate has on the value of p. He found that, if the time of closure is not greater than $2\,l \div v_p$, which is the time required for the pressure wave to make a "round trip" from the gate to the reservoir and back, then the full force of the maximum pressure p will be felt in the pipe. If a longer time t' be taken in moving the gate, then the pressure is reduced in intensity according to the proportion

$$\frac{p}{p_{max}} = \frac{t_r}{t'}. \qquad (141)$$

A very well written account of Joukovsky's experiments and deductions may be found in the 1904 Proceedings of the American Waterworks Association, p. 335. Under the head of Summary and Conclusions the author states, —

"The simplest method of protecting water pipes from water hammer is found in the use of slow-closing gates. The duration of closure should be proportional to the length of the pipe line. Air chambers, of adequate size, placed near the valves and gates, eliminate almost entirely the hydraulic shock, and do not allow the pressure wave to pass through them; but they must be very large, and it is difficult to keep them supplied with air. Safety valves allow to pass through them pressure waves of only such intensity as corresponds to the elasticity of the springs of the safety valves."

In the analytical discussion presented in this article the author has followed, with few exceptions, the method of deduction given by Professor Church in his "Hydraulic Motors," and its use here is with that writer's kind permission.

PROBLEMS

1. A nozzle having a coefficient of 0.90 delivers a jet 1 in. in diameter. What must be the amount of water discharged if the impulse of the jet be 150 lb.?

2. A jet from a 2-in. orifice is directed normally against a flat plate. What will be the dynamic pressure P when the discharge is 1 cu. ft. per second? Assume $c_c = 0.62$.

8. A horizontal tube, 8 ft. long and 2 in. in diameter, is filled with water under a pressure of 10 lb. per square inch and closed at the ends. If rotated in a horizontal plane about one end as an axis, at the rate of 60 R. P. M., what will the pressure at the outer end become?

Ans. 27.1 lb. per square inch.

9. A conical vessel with axis vertical and sides sloping at 30 degrees with the same is rotated about another axis distant 2 ft. from its own and parallel. How many revolutions per second must it make in order that water poured into it will be entirely discharged by the rotative effect?

Ans. 50.5 R. P. M.

CHAPTER IV

1. An 8-inch pipe contains a short section in which the diameter is gradually reduced to 3 in. and then gradually enlarged to full size. If the pressure of water passing through it is 75 lb. per square inch at a point just before the reduction commences, what will it be at the 3-inch section when the rate of flow is 1.20 cu. ft. per second? Assume no loss in head between the two sections considered.

Ans. 71.0 lb. per square inch.

2. A 3-inch pipe discharges into the air at a point 6 ft. above the level ground, the water leaving the pipe with a velocity of 28 ft. per second. Assuming air friction to be negligible, compute the velocity of the water as it strikes the ground if, (*a*) the pipe be horizontal; (*b*) the pipe be inclined upward 45 degrees from the horizontal.

Ans. 34.2 ft. per second.

3. A horizontal pipe 12 in. in diameter carries water with a mean velocity of 10 ft. per second. At a section, A, the pressure is 55 lb. per square inch and at a section, B, it is 40 lb. per square inch. Compute the quantity of energy passing each of these sections in 1 second, estimating the potential energy with reference to a datum plane through the pipe's axis. What is the amount of head lost between the two sections?

Ans. (*a*) 62,900 ft.-lb.
(*b*) 45,900 ft.-lb.
(*c*) 34.5 ft.

4. A 2-inch stream of water issues from a nozzle with a velocity of 75 ft. per second. What quantity of energy passes the nozzle per second if the datum plane for computing potential energy be taken through the axis of the issuing stream?

Ans. 9000 ft.-lb.

5. Water from a reservoir is pumped over a hill through a pipe 3 ft. in diameter, and a pressure of 30 lb. per square inch is maintained at the summit, where the pipe is 300 ft. above the reservoir. The quantity pumped is

49.5 cu. ft. per second and by reason of friction in the pump and pipe there is 10 ft. of head lost between reservoir and summit. What amount of energy must be furnished the water each second by the pump?

Ans. 2134 H. P.

6. How much energy, in horse-power units, is being transmitted through a 3-inch pipe in which the velocity of flow is 15 ft. per second and the gauge pressure 40 lb. per square inch? If a section farther on in the pipe were considered, would there be a less amount of energy passing each second? What form of energy would remain constant?

Ans. 8 H. P.

7. In order to maintain a discharge of 1.20 cu. ft. per second through a 6-inch pipe which discharges into the air, it is found necessary to keep a pressure of 36 lb. per square inch in the pipe at the inlet end, which is 8 ft. above the discharge point. Compute the loss in head while passing through the pipe. How much energy per second does it represent?

Ans. 90.8 ft.
6810 ft. lb. per second.

8. A 2-inch vertical pipe, supplied from a reservoir, discharges 0.22 cu. ft. per second into the air, and it is noted that its cross-section is filled by the water at exit. What will be the pressure in the pipe at a point 12 ft. above its lower end? Where will the pressure be absolute zero? What will it be at a point 40 ft. above the end? What fact is implied by the numerical value of the last result? Neglect friction in all cases.

Ans. (*a*) $p = 5.2$ lb. per square inch below atmospheric pressure.
(*b*) 34 ft. above end.

9. Water flows radially outward in all directions from between two horizontal circular plates which are 4 feet in diameter and placed parallel 1 inch apart. A supply of 1 cubic foot per second being maintained by a pipe entering one of the plates at its center, what pressure will exist between the plates at a point 6 inches from the center if no loss by friction be considered?

(NOTE. — The student may verify his answer by cutting two small disks from cardboard, piercing one at its center by a small pipe or piece of stiff straw, and blowing air into the pipe instead of water.)

Ans. $p = 0.1$ lb. per square inch below atmosphere.

10. Water enters a motor through a 4-inch pipe under a gauge pressure of 150 lb. per square inch. It leaves by an 8-inch pipe at an elevation 3 ft. below the point of entrance. If the pressure in the pipe at exit be 10 lb. per square inch and the discharge 2 cu. ft. per second, find the energy given up by the water each second as it passes the motor.

Ans. 41,600 ft.-lb.

11. During the test of a centrifugal pump, a gauge just outside the casing and on the 8-inch suction pipe registered a pressure 4 lb. per square

inch less than atmospheric. On the 6-inch discharge pipe another gauge indicated a pressure of 30 lb. per square inch above atmospheric.

If a vertical distance of 3 ft. intervened between the pipe centers at the sections where the gauges were attached, what horse power was expended by the pump in useful work when pumping 2 cu. ft. per second?

Ans. 18.7 H. P.

CHAPTER V

1. Compute the discharge in gallons per minute from a standard 2-inch circular orifice under a head of 6.784 ft.

Ans. 123.9 gal. per minute.

2. A circular orifice, 8 in. in diameter and having a thin edge, is made in the side of a reservoir which receives water at the rate of 6 cu. ft. per second. Determine the height above the center of the orifice to which water will rise in the reservoir, using proper coefficients from Table II.

Ans. 12.8 ft.

3. Water spurts vertically upward from an orifice in a horizontal plane under a head of 40 ft. What is the limit to the height of the jet if the coefficient of velocity be 0.97 and air friction negligible?

Ans. 37.6 ft.

4. A fluid of one fourth the density of water is discharged into the free atmosphere from an orifice in the side of a reservoir. If the pressure at a point in the reservoir on the same horizontal plane as the orifice be 50 lb. per square inch (absolute units), find the theoretic velocity of discharge.

Ans. 144.2 ft. per second.

5. Find the theoretic discharge through a vertical, sharp-edged orifice having the form of an isosceles triangle with an altitude of 10 inches, base (horizontal) 6 inches, and vertex level with the reservoir's surface. Disregard the velocity of approach.

Ans. 1.22 cu. ft. per second.

6. Water spurts out horizontally from a half-inch round hole in the side of a timber penstock, the wall of which is 2 in. thick. The jet strikes at a point 6 ft. horizontally distant from the orifice and 2 ft. lower down. Assuming the stream to issue from the orifice without contraction, due to the thickness of the wood penstock, compute the probable leakage in gallons per 24 hours.

Ans. 14,960 gal.

7. The jet from a circular, sharp-edged orifice, $\frac{1}{2}$ inch in diameter, under a head of 18 feet strikes a point distant 5 feet horizontally and 4.67 inches vertically from the center of the orifice. The discharge is 118.95

gallons in 569.2 seconds. Compute the coefficient of discharge, velocity, and contraction.

Ans. $C_d = 0.60.$
$C_v = 0.945.$
$C_c = 0.635.$

8. A steel box, rectangular in plan, floats with a draft of 2 ft. If the box be 20 ft. long, 10 ft. wide, and 6 ft. deep, compute the time necessary to sink it to its top edge by opening a standard orifice, 6 in. square, in its bottom. Neglect the thickness of the vertical sides.

Ans. 472 sec.

9. What will be the rate of discharge through a 1-inch orifice in the bottom of a vessel moving upward with an acceleration of 10 ft. per second each second, the water being 8 ft. deep over the orifice? Assume $C_d = 0.61$.

Ans. 0.087 cu. ft. per second.

10. A reservoir has a side wall, inclined backward 30 degrees from the vertical, in which is an orifice having a velocity coefficient of 0.97. With a head of 9 ft. on the orifice, compute

(*a*) Vertical height, above center of orifice, which jet will reach.
(*b*) Horizontal distance from orifice to jet.
(*c*) Velocity of jet as it strikes the ground a distance 7 ft. below orifice.

Ans. (*a*) 2.12 ft.
(*b*) 14.7 ft.
(*c*) 31.6 ft.

11. A standard 2-inch orifice connects two vessels, A and B. In A the water stands 4.6 ft. deep above the center of the orifice and its surface is exposed to steam at a pressure above atmospheric equivalent to 18 in. of mercury. In B the level of the water is 1 ft. above the orifice center and the air pressure on it is less than atmospheric by an amount corresponding to 10 in. of mercury. Compute the probable discharge in cubic feet per second.

Ans. 0.624 cu. ft. per second.

12. A sharp-edged orifice, 1 ft. square, discharges under a head of 1 ft., the latter being measured above the center of the orifice. Compute the rate of discharge assuming the mean head to be 1 ft., and compare the result with that obtained by using the exact formula in paragraph 43.

Ans. 4.81 cu. ft. per second.
4.75 cu. ft. per second.

13. A reservoir of water is connected with one containing oil by a sharp-edged circular orifice 2 in. in diameter. Determine the rate of discharge when the water stands 6 ft., and the oil 2 ft., deep on the orifice's center. The oil has a specific gravity of 0.80.

Ans. 0.22 cu. ft. per second.

14. A vessel 16 ft. high and 6 ft. in diameter contains water 7 ft. deep when at rest. How fast must it be revolved about a vertical axis through its center in order that the velocity (relative to the vessel) of flow from an orifice in the side and 1 ft. above the bottom, may be 24 ft. per second?

Ans. 65.8 R. P. M.

15. A rectangular tank with vertical sides, 16 ft. long and 4 ft. wide, contains water to a depth of 4 ft. How long will it take to empty it by opening a 4-inch, sharp-edged, circular orifice in its bottom, assuming a constant discharge coefficient of 0.60? *Ans.* 610 sec.

16. A canal lock 400 ft. long and 90 ft. wide is emptied through a submerged opening having an area of 60 sq. ft. and a discharge coefficient of 0.64. The water discharges into the lower canal, which is maintained at a constant level 20 ft. below that in the lock when full. How long will it take to bring the lock level down to that in the lower canal?

Ans. 17.4 min.

17. A reservoir, $\frac{1}{2}$ acre in area (21,780 sq. ft.), with sides nearly vertical so that it may be considered prismatic, receives a stream yielding 9 cu. ft. per second, and discharges through a sluice 4 ft. wide which is raised 2 ft. Calculate the time necessary to lower the surface 5 ft., the head over the center of the sluice, when opened, being 10 ft. Assume $c_d = 0.62$. *Ans.* 1104 sec.

18. The head in a vessel with vertical sides, at the instant of opening an orifice, was 9 ft. and at closing had decreased to 5 ft. Determine the constant head under which in the same time the orifice would discharge the same volume of water. *Ans.* 6.85 ft.

19. Water fills a vessel whose shape is that of an inverted pyramid with a square base. If the base measures 12 inches on a side and the altitude is 2 feet, how long a time will be required to empty the vessel through an orifice at the apex 1 inch square. Assume that $c_d = 0.88$ and disregard changes in dimensions of pyramid caused by cutting the orifice.

Ans. 11.6 sec.

20. From a given prismatic reservoir the discharge from an orifice at the base would be 32 cu. ft. in t seconds if the head were constant at 16 ft.

(*a*) Under a falling head, starting at 25 ft., how much must it fall during the same time, t, so that the discharge during that time shall also be 32 cu. ft., assuming the coefficient of discharge constant?

(*b*) What is the area of the vessel in square feet?

(*c*) If $c_d = 0.85$ and $t = 40$ seconds, what would be the area of the orifice?

Ans. Fall = 16 ft.
Area = 2 sq. ft.
$a = .029$ sq. ft.

21. If the actual velocity of flow from a certain orifice under an 8-foot head was found to be 22.09 ft. per second, what was the loss in head by friction?

Ans. 0.43 ft.

CHAPTER VI

1. A short cylindrical tube is attached to the vertical wall of a reservoir, but differs from a standard short tube in that it projects into the reservoir a distance of half its length. This causes the amount of the contraction at entrance to be increased and experimental observations indicate a coefficient of contraction at this point of 0.50. The coefficient of velocity for the whole tube being 0.72, determine what value the head on the tube must be in order that the pressure in the contracted vein shall be absolute zero. *Ans.* 30.6 ft.

2. Water is discharged from a reservoir through a standard short tube under a head of 30 ft.

(*a*) What is the velocity head in the stream at exit?
(*b*) What is the velocity head in the contracted section?
(*c*) What is the absolute pressure head at the contracted section?

Ans. (*a*) 20.2 ft.
(*b*) 49.5 ft.
(*c*) 11.5 ft.

3. A 2-inch nozzle is attached to a 6-inch pipe. Pressure at the base of the nozzle during the flow is 50 lb. per square inch. If the coefficient of velocity and discharge are both 0.90, find

(*a*) Velocity and rate of discharge.
(*b*) Energy per second in water discharged.

Ans. (*a*) 78 ft. per second.
1.72 cu. ft. per second.
(*b*) 10,150 ft.-lb. per second.

4. A $1\frac{1}{4}$-inch nozzle, attached to a $2\frac{1}{2}$-inch pipe, discharges 290 gal. per minute under a pressure of 40 lb. per square inch at the base of the nozzle. What is the coefficient of discharge of the nozzle?

Ans. 0.96.

5. A nozzle points vertically downward and terminates in a $1\frac{3}{8}$-inch orifice. It is supplied by a $2\frac{1}{2}$-inch pipe, to which is attached a pressure gauge 3 ft. above the nozzle orifice. When the gauge registers 30 lb. per square inch the discharge is found to be 310 gal. per minute. What head is being lost between the gauge and the orifice? Assume coefficient of contraction at exit to be 1.00.

Ans. 8.3 ft.

6. A $1\frac{1}{4}$-inch nozzle, attached to a horizontal $2\frac{1}{2}$-inch pipe, has a coefficient of discharge of 0.95. If, during flow, the gauge pressure in the pipe

be 80 lb. per square inch, what should be the discharge in gallons per minute? *Ans.* 398 gal. per minute.

7. A horizontal diverging tube, such as shown in Fig. 70, has a diameter of 3 in. at the contracted section and 4 in. at its mouth. Water stands 16 ft. deep above its axis on the reservoir side and 2 ft. deep on its discharging end. Determine the pressure existing in the contracted section during flow, assuming that 2 ft. of head is lost in the tube between the point of contraction and point of discharge.

Ans. 5.25 lb. per square inch abs.

8. If in the previous problem a vertical tube, 1 inch in diameter, were tapped into the contracted section and made to communicate with a second reservoir whose level was 8 ft. below the axis of the diverging tube, at what velocity would water flow from the vertical tube into the large one, assuming no change in pressure at the point of entrance?

Ans. 29.9 ft. per second.

9. A horizontal diverging tube connects two reservoirs. Its inlet end is rounded to render loss at entrance negligible and is 10 ft. below the water surface in the supplying reservoir. The diameter at the small section is 2 in. and the discharge end is 3 ft. below the second reservoir water level. What will be the proper value for the diameter of the discharge end consistent with maximum rate of flow through the tube? Assume a loss in head of 1 ft. in passing from the contracted section to the exit.

Ans. 2.86 in.

10. A horizontal diverging tube discharges water from a reservoir under a head of 8 feet. The diameter at the throat of the tube is 1 inch and at the exit 2 inches. If the velocity coefficient is 0.96 and the discharge is into air, what will be the pressure at the throat and how much head will be lost in the tube?

Ans. Lost head = 0.64 ft.
Absolute pressure = 5.44 lb. per square inch.

11. A horizontal diverging tube discharges water from a reservoir under a head of 6 ft. If the diameter at the large end be 4 inches and the discharge coefficient 0.96, what diameter will be necessary at the throat to produce a pressure there of one half an atmosphere?

Ans. 2.8 in.

12. A short reëntrant tube (similar to Borda's mouthpiece), 3 in. in diameter, is fitted in the vertical side of a reservoir whose superficial area is 22 sq. ft. What time will be required to lower the water level 2 ft. from an initial head of 8 ft.?

Ans. 83.3 sec.

CHAPTER VII

1. A weir with end contractions has a crest 10.37 ft. long and 2.87 ft. above the bottom of the channel. If the channel be 14.3 ft. wide, what

amount will be discharged by the weir under a head of 0.875 ft.? Use both Francis' and Bazin's formulæ and compare results, noting the magnitude of the head corresponding to the velocity of approach.

2. A suppressed weir, 6.97 ft. long, has its crest 2.79 ft. above the bottom of the channel. Compute the discharge under a head of 0.679 ft., using (*a*) Fteley and Stearns' formula; (*b*) Bazin's formula.

3. A rectangular channel 15 ft. wide contains water flowing 4 ft. deep with a mean velocity of 2.2 ft. per second. If a suppressed weir, 4.5 ft. high, be built across the channel, how much will the level of the water back of it be raised? Use Francis' formula, (1) neglecting velocity of approach, (2) with velocity of approach.

Ans. (*a*) 2.41 ft.
(*b*) 2.38 ft.

4. A triangular weir has a 90 degree notch. What head will be necessary to discharge 1000 gallons per minute?

5. A sharp-crested weir is to be built in a rectangular channel, 10 ft. in width, which discharges a maximum quantity of 8 cu. ft. per second. What length and crest height should the weir have in order that the head shall not exceed 8 in. or the depth of water back of the weir 3 ft.?

Ans. Length, 4.55 ft.
Crest height, 2.33 ft.

6. A reservoir whose area is 12,000 sq. ft. has an outlet through a contracted weir whose crest is 3 ft. long. How long a time will be required to lower the reservoir level 1 ft. from an initial head of 1.60 ft. on the weir crest? Use Hamilton Smith's formula and a mean value for the coefficient.

Ans. 20.9 min.

7. A reservoir, 50 ft. by 200 ft. in plan, has its sides vertical. It discharges through a rectangular suppressed weir, the initial head being 15 in. How long is the crest if 30 minutes are required to lower the level 13.50 in.?

Ans. 6.47 ft.

8. A rectangular channel has a maximum discharge of 160 cu. ft. per second when the water flows 4 ft. deep. If the width be 20 ft., how high should the crest of a submerged weir be in order that the water back of the weir shall not be over 5 ft. deep when the weir is built and discharging the given maximum flow?

Ans. 4.95 ft.

9. Given a submerged weir 9 ft. long with the upstream head 2.6 ft. and the downstream head 1.22 ft., compute the probable discharge.

Ans. 106 sec.-ft.

10. Compute the probable discharge through a trapezoidal weir having a crest length of 5 ft. and ends sloping at 30 degrees with the vertical. Assume the actual discharge to be 0.62 of the theoretical and compute for a head of 1 ft.

Ans. 17.5 sec.-ft.

11. A triangular weir has a 60 degree notch. Compute the discharge under a head of 1.6 ft., assuming a coefficient of 0.64.

Ans. 5.1 sec.-ft.

CHAPTER VIII

1. Find the rate of discharge from a 30-inch pipe, 3220 ft. long, supplied from a reservoir whose surface is 46 ft. above the pipe's end. Assume a clean, cast-iron pipe and make the computations (1) allowing for loss at entrance, (2) neglecting it.

Ans. (1) 56 cu. ft. per second.
(2) 56.6 cu. ft. per second.

2. Assuming a 12-inch pipe line, 2400 ft. long, to be made from wood staves, what will be its rate of discharge under a head of 32 ft.? What if the length were only 1200 ft.? ($f = .0175$.)

Ans. (1) 5.4 cu. ft. per second.
(2) 7.5 cu. ft. per second.

3. A 6-inch pipe suddenly enlarges to a diameter of 18 in., the velocity of flow in the 18-inch pipe being 2 ft. per second. Compute the lost head in feet and the foot-pounds of energy lost per second. What difference in pressure will be found in the two pipes near the enlargement? What would be the head lost if the velocity in the 18-inch pipe were 0.5 ft. per second?

Ans. (1) 3.98 ft.
(2) 884 ft.-lb.
(3) 0.43 lb. per square inch.
(4) 0.25 ft.

4. Two reservoirs with a difference in level of 90 ft. are connected by 2 mi. of 15-inch cast-iron pipe. Compute the rate of discharge.

Ans. 7.2 cu. ft. per second.

5. At what rate will water be discharged through 600 ft. of rubber-lined cotton hose, 3 in. in diameter, if it be attached to a hydrant at which the pressure is 60 lb. per square inch during flow? Assume no nozzle on the end of the hose and $f = 0.022$.

If a 1-inch nozzle be used, what discharge may be expected if a pressure of 70 lb. at the hydrant can be maintained? For the nozzle, $c_v = c_d = 0.96$? Assume hose horizontal and $f = 0.024$.

Ans. (1) 282 gal. per minute.
(2) 185 gal. per minute.

6. A 24-inch pipe is successively reduced in size to 12 in. and 6 in. the reductions being abrupt in each case. With a discharge of 2 cu. ft. per second, what total loss will be occasioned by the reductions? What

difference in pressure will there be in the 24- and 6-inch pipes if 2 ft. of head be lost by pipe friction in the 12-inch pipe?

Ans. (1) 0.55 ft.
(2) 1.8 lb. per square inch.

7. The pressure head at a point in a 12-inch cast-iron pipe is 50 ft. At a point 1000 ft. beyond, in the direction of flow, the pressure is 20 lb. per square inch. If the discharge be 5 cu. ft. per second, and 2 ft. of head is lost at intervening bends, what is the slope of the pipe?

Ans. 0.011.

8. Compute the diameter of pipe necessary for discharging 1500 gal. of water per minute, the pipe being of cast iron, 1000 ft. long, and its discharging end 4 ft. lower than the surface of the reservoir supplying it.

Ans. $d = 13.5$ in.

9. What head would be required for an 8-inch wood-stave pipe line, 3000 ft. long, leading from a reservoir and terminating in a 2-inch nozzle, the required discharge being that corresponding to a velocity of flow of 6 ft. per second in the pipe? Assume velocity coefficient for the nozzle at 0.95 and $f = 0.0223$.

Ans. 215 ft.

10. A level pipe line is abruptly enlarged from 4 to 8 in. in diameter. The velocity in the 4-inch pipe being 16 ft. per second, how much energy is wasted in heat at the enlargement? If the pressure head in the 4-inch pipe be 50 ft., what will it be in the 8-inch?

Ans. (1) 195 ft.-lb. per second.
(2) 51.49 ft.

11. An 18-inch cast-iron pipe is discharging 3000 gal. per minute. At a point 1000 ft. from the supplying reservoir (measured on pipe) the center of the pipe is 80 ft. below the reservoir surface. What pressure, in pounds per square inch, is to be expected there?

Ans. 33.4 lb. per square inch.

12. At a distance of 2000 ft. (measured on pipe) from the supplying reservoir, a 12-inch riveted steel pipe is 140 ft. below the reservoir's surface and the pressure is 50 lb. per square inch. What is the velocity in the pipe? Assume $f = .025$.

Ans. 6.2 ft. per second.

13. A 24-inch steel pipe line leaves a reservoir at elevation 1400 and runs 8000 ft. on a straight grade to elevation 1300; thence on a straight grade 4000 ft. to elevation 700, where it terminates in a 4-inch nozzle. The reservoir level being at elevation 1450, sketch the hydraulic grade line giving elevations at a sufficient number of points to define it. Assume $f = 0.025$ and $c_v = c_d = 0.95$ for the nozzle. At 80 % efficiency, what horse power can be developed by a wheel driven by the jet?

Ans. 960 H. P.

14. A pipe line, discharging into air, consists of two sections, one 500 ft. long and 12 inches in diameter, the other 1200 ft. long and 18 in. in diameter. If the change in section be abrupt and the quantity discharged be 3 cu. ft. per second, find the loss in head in each section due to pipe friction and the loss due to sudden enlargement. Plot the hydraulic grade line choosing suitable scales.

Ans. Lost heads are 2.46 ft., 0.77 ft., and 0.07 ft.

15. From a reservoir whose surface is at elevation 750, water is pumped through 4000 ft. of 12-inch pipe across a valley to a second reservoir whose level is at elevation 800. If, during pumping, the pressure is 80 lb. per square inch at a point on the pipe, midway of its length and at elevation 650, compute the rate of discharge and the power exerted by the pumps. Plot the hydraulic grade line.

Ans. 5.8 cu. ft. per second.
78.5 H. P.

16. A fire-engine with a 12-inch pump cylinder supplies water to a nozzle through 500 ft. of 3-inch hose. What power at the pump will be necessary to maintain a stream of water having a velocity of 75 ft. per second, with the nozzle held 30 ft. above the pump cylinder? The nozzle has a diameter of $1\frac{1}{2}$ in. at the tip and a coefficient of 0.90. The value of f for the hose may be assumed at 0.017.

Ans. 34 H. P.

17. A pump at elevation 900 is pumping 1.60 cu. ft. per second through 6000 ft. of 6-inch pipe to a reservoir whose level is at elevation 1250. What pressure will be found in the pipe at a point where the elevation is 1020 ft. above datum and the distance (measured along the pipe) from the pump 2500 ft.? Assume $f = .0225$.

Ans. 168 lb. per square inch.

18. Reservoir A is at elevation 1000 ft. above datum. Thence an 8-inch pipe line leads 3000 ft. to elevation 800, at which point it branches into two lines: a 6-inch line running 2000 ft. to reservoir B, elevation 850, and a 6-inch line running 1000 ft. to reservoir C, elevation 875. At what rate will water be delivered to each reservoir? Assume $f = 0.02$ in all cases.

Ans. To B, 1.35 cu. ft. per second.
To C, 1.45 cu. ft. per second.

19. A 12-inch pipe, 8000 ft. long, is connected with a reservoir whose surface is 250 ft. above the pipe's discharging end. If for the last 4000 ft. a second pipe of the same diameter be laid beside the first and connected to it, what would be the increase in discharge? Assume $f = 0.02$.

Ans. 2.1 cu. ft. per second.

20. Assuming the pipe line as described in the first part of the preceding problem, find the change in discharge resulting from inserting in the original line a section of 18-inch pipe 2000 ft. long. (No restriction is

made regarding the location of the enlarged portion.) Loss by change in section may be neglected and f assumed equal to 0.02.

Ans. 1.1 cu. ft. per second.

21. A pipe line 30,000 ft. long and 6 ft. in diameter supplies 10 nozzles with water from a reservoir whose level is 507 ft. above the nozzles. Each nozzle has an opening of 6 sq. in. and a coefficient of discharge and velocity of 0.95. Assuming $f = 0.017$, find the aggregate horse power available in the jets.

Ans. 3625 H. P.

22. A 6-inch pipe leaves a straight 4-inch pipe at a point A and later joins it again at a point C. The distance AC on the straight 4-inch pipe is 2000 ft. and on the 6-inch pipe it is 15,000 ft. How will the flow divide when it comes to A? Assume $f = 0.02$ for both pipes and consider only losses by pipe friction.

Ans. Ratio 1 to 1.

23. From a reservoir whose level is 300 ft. above a datum, a 12-inch pipe runs 12,000 ft. to a second reservoir whose level is at elevation 220 ft. A valve midway along the line is closed sufficiently to reduce the discharge to one half what it was with the valve wide open. The friction factor, f, may be taken as 0.023 with valve open and as 0.021 when it is partly shut. Compute the loss of head due to the partial closure of the valve, and sketch the hydraulic grade line. The loss at pipe entrance may be neglected, also that consumed in giving the water its velocity.

Ans. 61.7 ft.

24. At a point A, a 12-inch pipe is 400 ft. above a given datum. It terminates 6000 ft. beyond A, and at elevation 500, in the bottom of a standpipe which contains 30 ft. of water. With flow toward the standpipe, the hydraulic grade line at A is 150 ft. above the pipe. What is the rate of discharge into the standpipe if f be 0.02?

Ans. 2.6 cu. ft. per second.

25. Find the maximum power to be transmitted in a 36-inch pipe, the metal being $1\frac{1}{2}$ in. thick, the allowable stress 2800 lb. per square inch, and the velocity of flow 1 ft. per second. If the pipe is $1\frac{1}{4}$ mi. in length, find the loss of power by friction.

Ans. 434 H. P.; 0.47 H. P.

26. Two open, cylindrical tanks are connected by 1000 ft. of 3-inch iron pipe laid horizontally. Reservoir A is 25 ft. in diameter and its water level is 36 ft. above that in reservoir B whose diameter is 16 ft. How long, after opening a valve on the pipe line, will it be before the reservoir levels are the same? Assume f to be constant at 0.02 and neglect head lost at entrance.

Ans. 10 hr. 54 min.

27. A Venturi meter has an area ratio of 9 to 1, the larger diameter being 12 in. During flow the recorded pressure-head in the large section is 21.4 ft. and that at the throat, 13.9 ft. If C be 0.99, what rate of discharge through the meter is indicated?

Ans. 1.91 cu. ft. per second.

28. A 24-inch Venturi meter has a throat diameter of 8 in. During a 10-minute test it discharged 18,600 cu. ft. of water with a mean pressure-head at the large section of 112 ft. and at the throat a negative pressure corresponding to 10.50 in. of mercury. Compute the coefficient of the meter.

Ans. 0.992.

29. The discharge through a Venturi meter is 920 gallons per minute. The diameter of the pipe is 30 inches and the area ratio is 4 to 1. If the value of C be 0.99 and the pressure-head at entrance 18.8 feet, find the velocity and the pressure-head at the throat.

Ans. 18.76 ft.; 1.68 ft. per second.

30. The tip of the Pitot tube being placed at the center of a 24-inch pipe, the difference in the height of the velocity and pressure columns was found to be 22.3 inches. On the assumption that the curve of velocities was an ellipse, and that the side or surface velocity was one half that at the center, compute the discharge from the pipe. The tube's coefficient, c, was 1.05 in the formula, $h = c\dfrac{v^2}{2g}$.

Ans. 27.8 cu. ft. per second.

31. A Cole pitometer (see Art. 87) was used in conjunction with a differential gauge to measure the velocity in a 10-inch pipe. Carbon tetrachloride (spec. gr. 1.50) was used in the gauge and the deflection, z, noted as the pitometer tip was successively placed at 9 points on the pipe's diameter. Readings and corresponding distances from the pipe's axis follow:

Distance: 4.75″, 4.25″, 3.50″, 2.50″, 0.0″, 2.50″, 3.50″, 4.25″, 4.75″.

Deflection z: 2.72″, 3.96″, 5.15″, 5.61″, 5.90″, 5.86″, 5.35″, 4.48″, 3.68″.

The coefficient of the tube (see equation 95) was 1.42.

(*a*) Show that the velocity corresponding to any reading was $v = 0.84\sqrt{2\,g(s-1)z}$.

(*b*) Make a scale plot showing variation in velocity across the pipe.

(*c*) Compute discharge. Assume pipe's section to be divided by concentric rings so spaced that each point occupied by the pitometer falls midway between rings. Each annular area is to be multiplied by the mean of the velocities observed in it to give partial discharge.

(*d*) Compute pipe's coefficient, *i.e.* the ratio of mean velocity to center velocity.

32. The slope of the hydraulic grade line of a pipe is 0.005 and the pipe delivers 4000 gallons per minute. If the friction coefficient be assumed as that for a clean pipe plus a 50% increase to allow for roughening of surface with age, find the diameter of the pipe.

Ans. 20 in.

33. A velocity of 3 ft. per second is obtained through a 12-inch pipe whose hydraulic gradient slopes 3 ft. per 1000. Compute the value of the friction factor, f, and the amount of frictional resistance in pounds per square foot of pipe surface. *Ans.* $f = 0.0215$; $F = 30.2$.

34. A reservoir at elevation 300 ft. above datum furnishes water to a 24-inch pipe which leads to a point at elevation 100 ft., the pipe being 2000 ft. long. Here it branches into 3 pipes, 8 inches, 12 inches, and 6 inches in diameter. The 8-inch runs 1000 ft. and discharges at elevation 250, the 12-inch runs 1500 ft. to elevation 175, and the 6-inch runs 3000 ft. and discharges at elevation 100. Compute the discharge for each pipe. Assume $f = 0.02$.

Ans. From 8-in., 20.3 cu. ft. per second.
From 12-in., 10.9 cu. ft. per second.
From 6-in., 1.8 cu. ft. per second.

CHAPTER IX

1. A canal, in firm clean earth, has a bottom width of 20 ft. and side slopes of 2 horizontal to 1 vertical. If the water depth be 4 ft. and the slope of the surface 1 in 1500, compute the probable velocity and discharge.

Ans. 3.9 ft. per second.
437 cu. ft. per second.

2. A trapezoidal canal is to have a base width of 20 ft. and side slopes of 1 to 1. The nature of the soil (clean earth) limits the velocity of flow to 2 ft. per second. What slope must be given the bed in order to deliver 182 cu. ft. per second? *Ans.* $s = 0.00027$.

3. How deep will water flow in a 14-foot rectangular channel that is carrying 615 cu. ft. per second, if $s = 0.00075$ and the channel be lined with smooth cement?

Ans. 5.2 ft.

4. A triangular-shaped channel is to be designed to carry 24 cu. ft. per second on a slope of 0.0001. Determine what vertex angle and depth of water over the vertex will be necessary to give a section with minimum perimeter, assuming the channel to be built from timber planking.

Ans. 90 degrees; depth 3.6 ft.

5. A rectangular channel, 18 ft. wide and 4 ft. deep, has a slope of 1 in 1000, and is lined with good rubber masonry. ($n = 0.017$.) It is desired to increase as much as possible the amount discharged without changing the channel slope or form of section. The dimensions of the section may be changed but the channel must contain the same amount of lining as the old. Compute the new dimensions and probable increase in discharge.

Ans. 13 ft. by 6.5 ft.; 118 cu. ft. per second.

6. A rectangular channel, 12 ft. wide and 4 ft. deep, is lined with smooth stone, well laid, and has a hydraulic slope of 0.001. What saving in earth

excavation and lining, per foot of length, could have been effected by using more favorable dimensions?

Ans. New channel 9.8 ft. by 4.9 ft.

7. A canal is to have a trapezoidal section with one side vertical and the other sloping at 45 degrees. It is to carry 900 cu. ft. per second with a mean velocity of 3 ft. per second. Compute the dimensions of the section which would require a minimum hydraulic slope.

Ans. Base width 17.7 ft.; depth 12.5 ft.

8. In designing a canal for supplying a power plant the problem arises as to whether a trapezoidal or rectangular section should be built. If the former be used, it is found that, with a clean earth lining, a section having a top width of 40 ft., bottom width 10 ft., and water depth 5 ft., will deliver the required quantity of water if laid on a slope of 0.75 per 1000. Could a less slope be used by employing a rectangular section, lined with rubble masonry, if the area and velocity of flow be maintained at the same figures as in the trapezoidal section? What would its value be?

Ans. $s =$ ~~0.00052~~

9. Two circular conduits, each 5 ft. in diameter, serve to carry the waters of a creek through a railroad embankment. When carrying flood discharges both ends of the conduits are submerged. Assuming no change in the hydraulic gradient or in the value of C, what width would be necessary in two equal rectangular sections, each 4 ft. deep, if they are to replace the circular conduits and perform the same service? *Ans.* 5.2 ft.

10. Two ponds with a difference in level of 0.50 ft. are connected by 4000 ft. of rectangular canal, lined with smooth cement. The canal is 20 ft. wide, laid on uniform grade, with water 8 ft. deep at its upper end and 10 ft. at the lower end. Compute the rate of discharge.

Ans. 810 cu. ft. per second.

11. A rectangular canal 6000 ft. long, 10 ft. wide, is carrying 80 cu. ft. per second. The canal floor is level and at its lower end is fitted with a suppressed weir 3.5 ft. high. Assuming $n = 0.017$ and that the stream surface is a plane, compute the depth of water necessary in the canal at its upper end in order to maintain the given discharge.

Ans. 5.8 ft.

12. In a trapezoidal channel in clean earth, 12 ft. wide on the bottom, side slopes 2 horizontal to 1 vertical, and with a longitudinal bed slope of 1 to 500 (measured as the sine of the inclination angle), water is running 3 ft. deep. With constant flow maintained, a weir is introduced at a point so that the water depth just back of the weir is increased to 5 ft. At what distance upstream should a depth of 4.5 ft. be found?

(*a*) Solve by method of Art. 99.
(*b*) Solve by Bresse's formula (113) and table.

Ans. (*a*) 280 ft.; (*b*) 294 ft.

13. A brick-lined sewer, 3 ft. in diameter, is built on a constant grade of 1 in 1000. What quantity will it discharge when flowing full but under no pressure? What if flowing half full?

Ans. 21.2 cu. ft. per second.
10.6 cu. ft. per second.

CHAPTER X

1. A jet of water 1 in. in diameter exerts a pressure of 150 lb. against a flat plate held normal to the stream's axis. Compute the rate of discharge.

Ans. 0.65 cu. ft. per second.

2. A jet from a 2-inch orifice is directed normally against a flat plate. What pressure will it exert on the plate when $Q = 1$ cu. ft. per second?

Ans. 144 lb.

3. In approaching a bridge, a water main, 2 ft. in diameter, curves, from a horizontal position, upward through 45 degrees. What vertical component of dynamic pressure is developed in the bend under a velocity of 6 ft. per second? *Ans.* 155 lb.

4. A jet 2 in. in diameter, having a velocity of 140 ft. per second, is deflected by a curved surface through an angle of 45 degrees. Compute the value of the components of the normal pressure developed which are perpendicular and parallel respectively to the initial direction of the jet. Also compute the parallel component for a deflection of 90 degrees and 180 degrees.

Ans. (*a*) 586 lb.; 243 lb.
(*b*) 829 lb.
(*c*) 1658 lb.

5. A 2-inch jet from a nozzle is deflected through an angle of 175 degrees by striking against a curved vane which moves in the same direction as the jet and with a velocity of 100 ft. per second. What pressure will be on the vane when the nozzle is discharging 6 cu. ft. per second? What if the vane were stationary?

Ans. (*a*) 2590 lb.; (*b*) 6390 lb.

6. Compute the force exerted (in an axial direction) on a pipe by water which is flowing through it with a velocity of 8 ft. per second. The diameter is 1 ft., length 1000 ft., and f may be assumed as 0.02.

Ans. 984 lb.

7. A 2-inch nozzle is attached to a 12-inch pipe by flange bolts. What will be the total stress in the bolts when the pressure at the base of the nozzle is 75 lb. per square inch? Assume $c_d = c_v = 0.96$.

Ans. 8050 lb.

8. A 36-inch pipe is changed by gradual reduction to a 24-inch diameter. Before entering the reducer the water has a velocity of 7 ft. per second and

3. Find the reaction of a jet from a 3-in. nozzle discharging 10 cu. ft. per second, the jet suffering no contraction as it leaves the nozzle.

4. In approaching a bridge, a water main, 2 ft. in diameter, curves from a horizontal position upward through 45°. What vertical component of dynamic pressure is developed in the bend under a velocity of flow of 6 ft. per second?

5. A 2-in. jet from a nozzle is deflected through 175° by striking against a vane moving in the same direction as the jet with a velocity of 100 ft. per second. What is the dynamic pressure upon the vane when the nozzle is discharging 6 cu. ft. per second? What would be the pressure if the vane were stationary?

6. The buckets of an impulse wheel receive water from a 2-in. nozzle from which the jet issues with a volocity of 60 ft. per second. If the speed of the wheel is such that the velocity of the buckets is 40 ft. per second, and the angle α be 170°, find the total turning force applied to the wheel. At what speed should the wheel be run to make the power a maximum? What would it then be?

7. A 6-ft. pipe conducting water to a number of turbines is made from steel plate 0.25 in. thick. When the velocity of flow is 8 ft. per second, a quick-closing gate operates to stop the flow. What is the excess pressure tending to burst the pipe?

LITERATURE

Water Hammer

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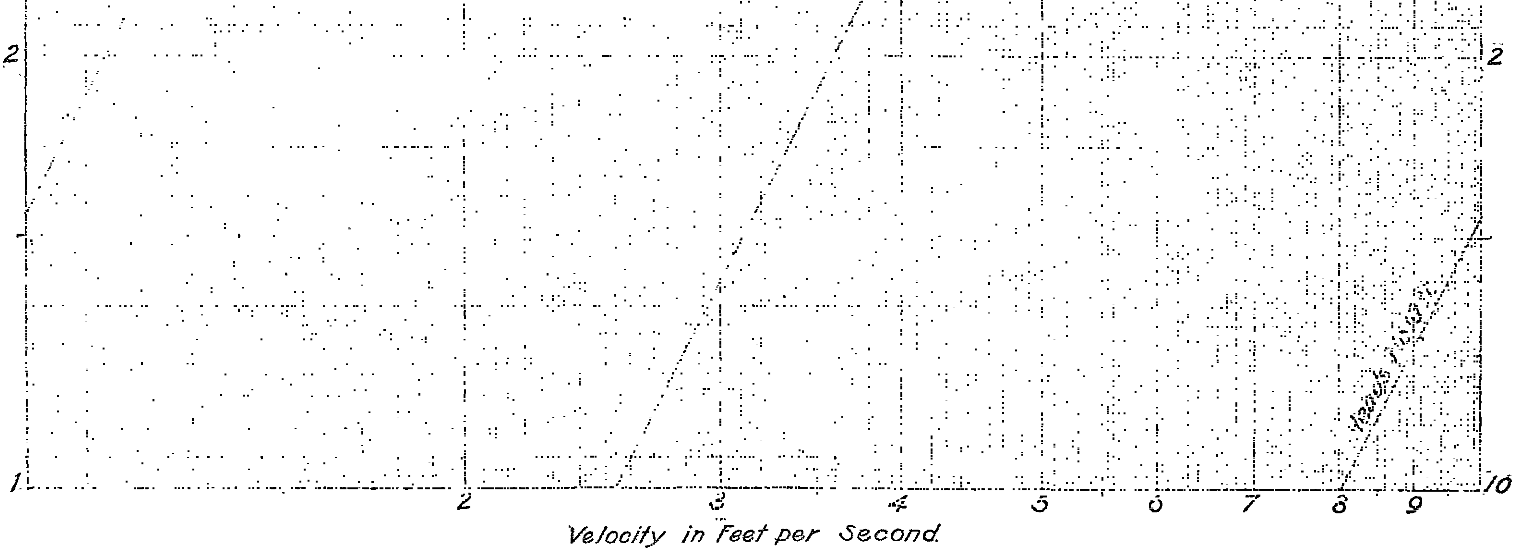
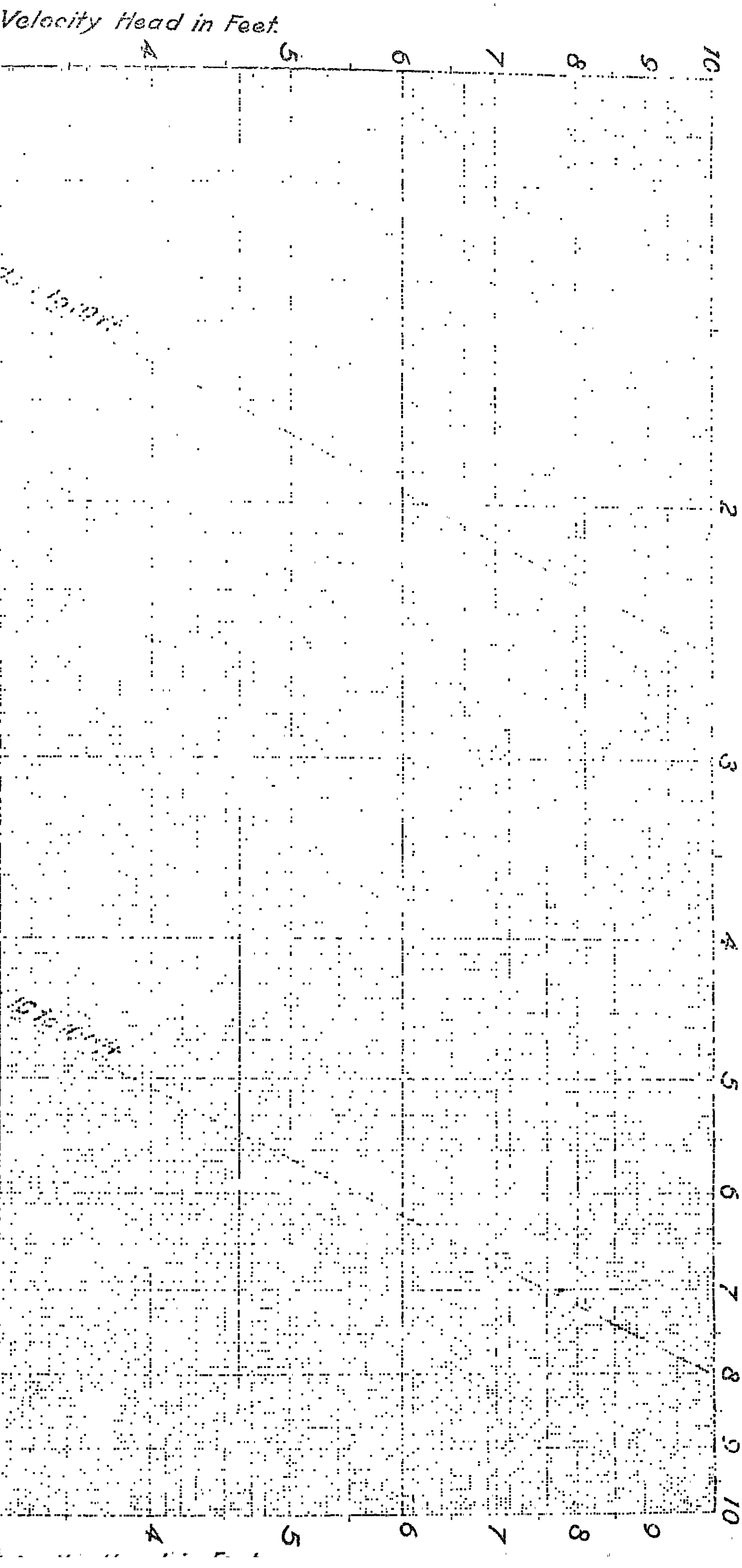


CONVERSION DIAGRAM

VELOCITY v. VELOCITY HEAD

$$\left(\frac{v^2}{2g} = h\right)$$

NOTE.—The marginal figures represent the first significant figure only, and the position of the decimal point must be determined by inspection.


Velocity Head in Feet
4
5
6
7
8
9
10
2
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10

APPENDIX

TABLE I

Weight of Distilled Water

From Hamilton Smith's Hydraulics

...rature ...enheit.	Pounds per Cubic Foot.	Temperature Fahrenheit.	Pounds per Cubic Foot.	Temperature Fahrenheit.	Pounds per Cubic Foot.
...2	62.416	95	62.061	160	61.006
...5	62.421	100	61.998	165	60.904
...9.3	62.424	105	61.933	170	60.799
...5	62.419	110	61.865	175	60.694
...0	62.408	115	61.794	180	60.586
...5	62.390	120	61.719	185	60.476
...0	62.366	125	61.638	190	60.365
...5	62.336	130	61.555	195	60.251
...0	62.300	135	61.473	200	60.135
...5	62.261	140	61.386	205	60.015
...0	62.217	145	61.296	210	59.893
...5	62.169	150	61.203	212	59.843
...0	62.118	155	61.106		

TABLE II

Coefficients of Discharge (c) for Circular Orifices

From Hamilton Smith's Hydraulics

...ↄ ...et.	Diameter of Orifice in Feet.						
	0.02	0.04	0.07	0.1	0.2	0.6	1.0
...4		0.637	0.624	0.618			
...6	0.655	.630	.618	.613	0.601	0.593	
...8	.648	.626	.615	.610	.601	.594	0.590
...0	.644	.623	.612	.608	.600	.595	.591
...5	.637	.618	.608	.605	.600	.596	.593
...0	.632	.614	.607	.604	.599	.597	.595
...5	.629	.612	.605	.603	.599	.598	.596
...0	.627	.611	.604	.603	.599	.598	.597
...0	.623	.609	.603	.602	.599	.597	.596
...0	.618	.607	.602	.600	.598	.597	.596
...0	.614	.605	.601	.600	.598	.596	.596
...0	.611	.603	.599	.598	.597	.596	.595
...0	.601	.599	.597	.596	.596	.596	.594
...0	.596	.595	.594	.594	.594	.594	.593
...0	.593	.592	.592	.592	.592	.592	.592

TABLE III

Coefficients of Discharge (c) for Square Orifices

From Hamilton Smith's Hydraulics

Head h in Feet.	Side of the Square in Feet.						
	0.02	**0.04**	**0.07**	**0.1**	**0.2**	**0.6**	**1.0**
0.4		0.643	0.628	0.621			
0.6	0.660	.636	.623	.617	0.605	0.598	
0.8	.652	.631	.620	.615	.605	.600	0.597
1.0	.648	.628	.618	.613	.605	.601	.599
1.5	.641	.622	.614	.610	.605	.602	.601
2.0	.637	.619	.612	.608	.605	.604	.602
2.5	.634	.617	.610	.607	.605	.604	.602
3.0	.632	.616	.609	.607	.605	.604	.603
4.0	.628	.614	.608	.606	.605	.603	.602
6.0	.623	.612	.607	.605	.604	.603	.602
8.0	.619	.610	.606	.605	.604	.603	.602
10.0	.616	.608	.605	.604	.603	.602	.601
20.0	.606	.604	.602	.602	.602	.601	.600
50.0	.602	.601	.601	.600	.600	.599	.599
100.0	.599	.598	.598	.598	.598	.598	.598

TABLE IV

Coefficients of Discharge (c) for Rectangular Orifices 1 Foot Wide

From Fanning's Treatise on Hydraulic and Water Supply Engineering

Head h in Feet.	Depth of Orifice in Feet.						
	2.0	**1.5**	**1.0**	**0.75**	**0.50**	**0.25**	**0.125**
0.4						0.619	0.625
0.6					0.614	.618	.623
0.8				0.609	.612	.616	.622
1.0		0.608	0.605	.608	.611	.616	.622
1.5	0.609	.607	.604	.607	.610	.614	.620
2.0	.609	.606	.604	.606	.609	.613	.619
2.5	.608	.606	.603	.606	.609	.613	.617
3.0	.607	.605	.603	.605	.608	.612	.616
4.0	.606	.604	.602	.604	.606	.610	.614
6.0	.604	.603	.601	.602	.604	.608	.610
8.0	.603	.602	.601	.601	.603	.605	.608
10.0	.601	.601	.601	.601	.602	.604	.606
15.0	.602	.601	.601	.601	.601	.603	.607
20.0	.602	.601	.601	.601	.602	.604	.607

TABLE V

Coefficients of Discharge (c) for Submerged Orifices

Based on Data from Hamilton Smith's Hydraulics

Effective Head in Feet.	Size of Orifice in Feet.				
	Circle **0.05**	Square **0.05**	Circle **0.1**	Square **0.1**	Rectangle **0.05 × 0.3**
0.5	0.615	0.619	0.603	0.608	0.623
1.0	.610	.614	.602	.606	.622
1.5	.607	.612	.600	.605	.621
2.0	.605	.610	.599	.604	.620
2.5	.603	.608	.598	.604	.619
3.0	.602	.607	.598	.604	.618
4.0	.601	.606	.598	.604	

TABLE VI

Velocity Head Coefficients for Use in Fteley and Stearns' Weir Formula

(For use, see Art. 63.)

Taken from Fteley and Stearns' Original Paper

Head on Weir.	Depth of Channel of Approach below Crest.			
	2.60	**1.70**	**1.00**	**0.50**
0.20	1.51	1.66	1.87	1.70
0.30	1.50	1.65	1.83	1.53
0.40	1.49	1.63	1.79	1.53
0.50	1.48	1.62	1.75	1.53
0.60	1.47	1.60	1.71	1.52
0.70	1.46	1.59	1.68	1.51
0.80	1.45	1.57	1.65	1.50
0.90	1.44	1.56	1.63	1.49
1.00	1.43	1.54	1.61	1.48
1.10	1.42	1.53	1.59	
1.20	1.41	1.51	1.57	
1.30	1.40	1.49	1.55	
1.40	1.39	1.48	1.54	
1.50	1.38	1.46	1.52	
1.60	1.37	1.44	1.51	
1.70	1.36	1.43	1.49	
1.80	1.35	1.41		
1.90	1.34	1.40		
2.0	1.33	1.38		

TABLE VII

Coefficient (c) for Contracted Weirs

From Hamilton Smith's Hydraulics

Effective Head in Feet.	Length of Weir in Feet.						
	0.66	1	2	3	5	10	19
0.1	0.632	0.639	0.646	0.652	0.653	0.655	0.656
0.15	.619	.625	.634	.638	.640	.641	.642
0.2	.611	.618	.626	.630	.631	.633	.634
0.25	.605	.612	.621	.624	.626	.628	.629
0.3	.601	.608	.616	.619	.621	.624	.625
0.4	.595	.601	.609	.613	.615	.618	.620
0.5	.590	.596	.605	.608	.611	.615	.617
0.6	.587	.593	.601	.605	.608	.613	.615
0.7		.590	.598	.603	.606	.612	.614
0.8			.595	.600	.604	.611	.613
0.9			.592	.598	.603	.609	.612
1.0			.590	.595	.601	.608	.611
1.2			.585	.591	.597	.605	.610
1.4			.580	.587	.594	.602	.609
1.6				.582	.591	.600	.607

TABLE VIII

Coefficients (c) for Suppressed Weirs

From Hamilton Smith's Hydraulics

Effective Head in Feet.	Length of Weir in Feet.							
	0.66	2	3	4	5	7	10	19
0.1	0.675				0.659	0.658	0.658	0.657
0.15	.662	0.652	0.649	0.647	.645	.645	.644	.643
0.2	.656	.645	.642	.641	.638	.637	.637	.635
0.25	.653	.641	.638	.636	.634	.633	.632	.630
0.3	.651	.639	.636	.633	.631	.629	.628	.626
0.4	.650	.636	.633	.630	.628	.625	.623	.621
0.5	.650	.637	.633	.630	.627	.624	.621	.619
0.6	.651	.638	.634	.630	.627	.623	.620	.618
0.7	.658	.640	.635	.631	.628	.624	.620	.618
0.8	.656	.643	.637	.633	.629	.625	.621	.618
0.9		.645	.639	.635	.631	.627	.622	.619
1.0		.648	.641	.637	.633	.628	.624	.619
1.2			.646	.641	.636	.632	.626	.620
1.4				.644	.640	.634	.629	.622
1.6				.647	.642	.637	.631	.623

TABLE IX

Values of Friction Factor (f) for Clean Iron Pipes

Based on Data from Fanning.

Diameter in Inches.	Velocity in Feet per Second.								
	0.5	1	2	3	4	6	10	15	20
1	.0398	.0353	.0317	.0299	.0285	.0266	.0244	.0232	.0228
2	.0364	.0330	.0300	.0284	.0272	.0255	.0237	.0226	.0222
3	.0354	.0316	.0288	.0273	.0262	.0248	.0232	.0222	.0218
6	.0317	.0289	.0264	.0252	.0243	.0231	.0219	.0211	.0208
8	.0296	.0275	.0252	.0242	.0233	.0223	.0212	.0206	.0202
10	.0283	.0262	.0242	.0232	.0225	.0216	.0206	.0200	.0197
12	.0268	.0251	.0233	.0224	.0218	.0209	.0201	.0195	.0192
18	.0238	.0224	.0211	.0204	.0199	.0193	.0188	.0183	.0181
24	.0212	.0194	.0193	.0187	.0184	.0180	.0176	.0172	.0170
30	.0194	.0186	.0179	.0175	.0173	.0170	.0166	.0162	.0161
36	.0177	.0172	.0167	.0164	.0162	.0160	.0156	.0154	.0152
48	.0153	.0150	.0147	.0145	.0144	.0143	.0141	.0139	.0138
60	.0137	.0135	.0133	.0132	.0131	.0130	.0128	.0126	.0125
72	.0125	.0124	.0122	.0120	.0120	.0118	.0117	.0117	.0117
96	.0109	.0107	.0106	.0106	.0105	.0105	.0104	.0104	.0103

TABLE X

Coefficient (c) in Kutter's Formula

(See Article 93.)

Slope	n	Hydraulic Radius r in Feet.														
		0.2	0.3	0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0	4.0	6.0	8.0	10.0	15.0
.00005	.010	87	98	109	123	133	140	154	164	172	177	187	199	207	213	220
	.012	68	78	88	98	107	113	126	135	142	148	157	168	176	182	189
	.015	52	58	66	76	83	89	99	107	113	118	126	138	145	150	159
	.017	43	50	57	65	72	77	86	93	98	103	112	122	129	134	142
	.020	35	41	45	53	59	64	72	80	84	88	95	105	111	116	125
	.025	26	30	35	41	45	49	57	62	66	70	78	85	92	96	104
	.030	22	25	28	33	37	40	47	51	55	58	65	74	78	83	90
.0001	.010	98	108	118	131	140	147	158	167	173	178	186	196	202	206	212
	.012	76	86	95	105	113	119	130	138	144	148	155	165	170	174	180
	.015	57	64	72	81	88	93	103	109	114	118	125	134	140	143	150
	.017	48	55	62	70	75	80	88	95	99	104	111	118	125	128	135
	.020	38	45	50	57	63	67	75	81	85	88	95	102	107	111	118
	.025	28	34	38	43	48	51	59	64	67	70	77	84	89	93	98
	.030	23	27	30	35	39	42	48	52	55	59	64	72	75	80	85
.0002	.010	105	115	125	137	145	150	162	169	174	178	185	193	198	202	206
	.012	83	92	100	110	117	123	133	139	144	148	154	162	167	170	175
	.015	61	69	76	84	91	96	105	110	114	118	124	132	137	140	145
	.017	52	59	65	73	78	83	90	97	100	104	110	117	122	125	130
	.020	42	48	53	60	65	68	76	82	85	88	94	100	105	108	113
	.025	30	35	40	45	50	54	60	65	68	70	76	83	86	90	95
	.030	25	28	32	37	40	43	49	53	56	59	63	69	74	77	82
.0004	.010	110	121	128	140	148	153	164	171	174	178	184	192	197	198	203
	.012	87	95	103	113	120	125	134	141	145	149	153	161	165	168	172
	.015	64	73	78	87	93	98	106	112	115	118	123	130	134	137	142
	.017	54	62	68	75	80	84	92	98	101	104	110	116	120	123	128
	.020	43	50	55	61	67	70	77	83	86	88	94	99	104	106	110
	.025	32	37	42	47	51	55	60	65	68	70	75	82	85	88	92
	.030	26	30	33	38	41	44	50	54	57	59	63	68	73	75	80
.001	.010	113	124	132	143	150	155	165	172	175	178	184	190	195	197	201
	.012	88	97	105	115	121	127	135	142	145	149	154	160	164	167	171
	.015	66	75	80	88	94	98	107	112	116	119	123	130	133	135	141
	.017	55	63	68	76	81	85	92	98	102	105	110	115	119	122	127
	.020	45	51	56	62	68	71	78	84	87	89	93	98	103	105	109
	.025	33	38	43	48	52	55	61	65	68	70	75	81	84	87	91
	.030	27	30	34	38	42	45	50	54	57	59	63	68	72	74	78
.01	.010	114	125	133	143	151	156	165	172	175	178	184	190	194	196	200
	.012	89	99	106	116	122	128	136	142	145	149	154	159	163	166	170
	.015	67	76	81	89	95	99	107	113	116	119	123	129	133	135	140
	.017	56	64	69	77	82	86	93	99	103	105	109	115	118	121	126
	.020	46	52	57	63	68	72	78	84	87	89	93	98	102	105	108
	.025	34	39	44	49	52	56	62	65	68	70	75	80	83	86	90
	.030	27	31	35	39	43	45	51	55	58	59	63	67	71	73	77

TABLE XI

Coefficients (c) in Bazin's Formula

(See Article 94)

Hydraulic Radius r in Feet	Coefficient of Roughness m.					
	m = .06	m = .16	m = .46	m = .85	m = 1.30	m = 1.75
0.2	126	96	55	36	25	19
.3	132	103	63	41	30	23
.4	134	108	68	46	33	26
.5	136	112	71	50	36	29
.75	140	118	80	57	42	34
1.0	142	122	86	62	47	38
1.25	143	125	90	66	51	41
1.50	145	127	94	70	54	44
2.0	146	131	99	76	59	49
2.5	147	133	104	80	63	53
3.0	148	135	106	83	67	57
5.0	150	140	115	93	77	65
7.0	152	142	120	100	83	72
10·0	152	144	125	106	91	79
12.0	153	145	127	109	94	82
16.0	153	147	130	114	99	88
20.0	154	148	133	117	103	92

TABLE XII

Areas of Circles

Diameter in Inches	Area: Square Inches	Area: Square Feet	Diameter in Inches	Area: Square Inches	Area: Square Feet	Diameter in Inches	Area: Square Inches	Area: Square Feet
1.0	0.7854	.005454	4.0	12.5664	.087266	7.0	38.4845	.267254
.1	0.9503	.006599	.1	13.2025	.091684	.1	39.5919	.274944
.2	1.1310	.007854	.2	13.8544	.096211	.2	40.7150	.282743
.25	1.2272	.008522	.25	14.1863	.098516	.25	41.2825	.286684
.3	1.3273	.009218	.3	14.5220	.100847	.3	41.8539	.290652
.4	1.5394	.010690	.4	15.2053	.105592	.4	43.0084	.298669
.5	1.7671	.012272	.5	15.9043	.110446	.5	44.1786	.306796
.6	2.0106	.013963	.6	16.6190	.115410	.6	45.3646	.315032
.7	2.2698	.015763	.7	17.3494	.120482	.7	46.5663	.323377
.75	2.4053	.016703	.75	17.7205	.123059	.75	47.1730	.327590
.8	2.5447	.017671	.8	18.0956	.125664	.8	47.7836	.331831
.9	2.8353	.019689	.9	18.8574	.130954	.9	49.0167	.340394
2.0	3.1416	.021816	5.0	19.6350	.136354	8.0	50.2655	.349066
.1	3.4636	.024053	.1	20.4282	.141863	.1	51.5300	.357847
.2	3.8013	.026398	.2	21.2372	.147480	.2	52.8102	.366737
.25	3.9761	.027612	.25	21.6475	.150330	.25	53.4562	.371223
.3	4.1548	.028852	.3	22.0618	.153207	.3	54.1061	.375736
.4	4.5239	.031416	.4	22.9022	.159043	.4	55.4177	.384845
.5	4.9087	.034088	.5	23.7583	.164988	.5	56.7450	.394063
.6	5.3093	.036870	.6	24.6301	.171042	.6	58.0880	.403389
.7	5.7256	.039760	.7	25.5176	.177205	.7	59.4468	.412825
.75	5.9396	.041247	.75	25.9672	.180328	.75	60.1320	.417584
.8	6.1575	.042760	.8	26.4208	.183477	.8	60.8212	.422370
.9	6.6052	.045869	.9	27.3397	.189859	.9	62.2114	.432024
3.0	7.0686	.049087	6.0	28.2743	.196350	9.0	63.6173	.441786
.1	7.5477	.052414	.1	29.2247	.202949	.1	65.0388	.451658
.2	8.0425	.055851	.2	30.1907	.209658	.2	66.4761	.461640
.25	8.2958	.057609	.25	30.6796	.213053	.25	67.2006	.466671
.3	8.5530	.059396	.3	31.1725	.216475	.3	67.9291	.471730
.4	9.0792	.063050	.4	32.1699	.223402	.4	69.3978	.481929
.5	9.6211	.066813	.5	33.1831	.230438	.5	70.8822	.492237
.6	10.1788	.070686	.6	34.2119	.237583	.6	72.3823	.502654
.7	10.7521	.074667	.7	35.2565	.244837	.7	73.8981	.513181
.75	11.0447	.076699	.75	35.7847	.248505	.75	74.6619	.518486
.8	11.3411	.078758	.8	36.3168	.252200	.8	75.4296	.523817
.9	11.9459	.082958	.9	37.3928	.259672	.9	76.9769	.534561

The above table may be used for finding the areas of circles whose diameters are not within the limits of the table. Since the areas vary as the squares of their diameters, the given diameter may be divided (or multiplied) by 10, and the area found from the table under the resulting diameter corrected by moving the decimal point two places to the right (or left). Thus to find the area of a 22-inch circle:

From table, area of 2.2-inch circle = 3.8013 sq. in. = .026398 sq. ft.
Therefore area of 22-inch circle = 380.13 sq. in. = 2.64 sq. ft.

Again, to find the area of a 0.75-inch circle:

From table, area of 7.5-inch circle = 44.1786 sq. in = 0.306796 sq. ft.
Therefore area of 0.75-inch circle = 0.4418 sq. in. = 0.00307 sq. ft.

It will also be apparent that the *first two* columns in the table may be used for any unit of measure.

REVISED PROBLEMS

The following problems are presented as a revision of those appearing at the ends of previous chapters. In a majority of cases the author has given the numerical answers, believing them to be a great help to the student in checking his work. Pains have been taken to have them correct; but, owing to the large number of calculations involved, errors may have been overlooked to which the author would appreciate having his attention called.

CAMBRIDGE, MASS.
August 16, 1918.

CHAPTER I

1. If a vertical cylindrical column of water, 100 ft. high, be contained in a steel reservoir, compute its probable height, if water were absolutely incompressible.

Ans. 100.012 ft.

CHAPTER II

1. A gas holder contains illuminating gas under a pressure corresponding to 2 in. of water. If the holder be at sea level (atmospheric pressure 14.7 lb. per square inch), what pressure in inches of water may be expected in one of its distributing pipes at a point 500 ft. above sea level? Assume unit weights of air and gas to be constant at all elevations with values of 0.08 and 0.04 lb. per cubic foot respectively.

Ans. 5.84 in.

2. A rectangular plate is immersed vertically with one of its sides in the water surface. How must a straight line be drawn from one end of that side so as to divide the rectangle into two areas, the total pressure upon which shall be equal?

3. A vertical gate, 4 ft. wide and 6 ft. high, hinged at the upper edge, is kept closed by the pressure of water standing 8 ft. deep over its top edge. What force applied normally at the bottom of the gate would be required to open it?

Ans. 9000 lb.

4. A vertical triangular plate whose height is 12 ft. has its base horizontal and vertex uppermost in the water's surface. Find the depth to which it must be lowered so that the difference in level between the center of gravity and the center of pressure shall be 8 in.

Ans. 4 ft.

5. Find the depth to the center of pressure on a trapezoidal surface, vertically immersed in water, the upper base being 5 ft. long, parallel to and 10 ft. below the water surface. The trapezoid is symmetrical about a vertical center line, its lower base being 3 ft. long and 13 ft. below the surface.

Ans. 11.44 ft.

6. A flat parabolic plate is immersed, with its axis vertical, in water until its vertex is 7 ft. below the water surface. Locate the center of pressure.

Ans. 4.0 ft. down.

7. Compute the position of the center of pressure on a circular gate, 4 ft. in diameter, placed with its center 4 ft. below the water surface and in a plane inclined 45 degrees from the vertical.

Ans. 0.18 ft. below center of gravity.

8. A plate shaped as a right triangle is immersed in water with one side vertical. If the head on the upper vertex be 3 ft., the length of the vertical side, 6 ft., and that of the horizontal side, 10 ft., locate the center of pressure. Find its lateral position by the calculus method and check by means of medial line.

Ans. 3.57 ft. below center of gravity.
0.29 ft. from vertical side.

9. An isosceles triangle, base 10 ft. and altitude 20 ft., is immersed vertically in water with its axis of symmetry horizontal. If the head on its axis be 30 ft., locate the center of pressure both laterally and vertically.

Ans. $x_c = 30.14$ ft.
$y_c = 6.67$ ft. from base.

10. A vertical, rectangular sluice gate at the bottom of a dam is 2 ft. wide, 6 ft. high, and exposed to water pressure on one side corresponding to a head of 50 ft. above its center. Assuming the gate and stem to weigh 500 lb. and the coefficient of friction of gate on runners to be 0.25, find the force necessary to raise it.

Ans. 9875 lb.

11. A tank with plane vertical sides contains 4 ft. of mercury and 12 ft. of water. Find the total pressure on a portion of a side, 1 ft. square, one half this portion being below the surface of the mercury. The sides of the square area are vertical and horizontal.

Ans. 849 lb.

12. A triangle having a base of 4 ft. and an altitude of 6 ft. is wholly immersed in water, its base being in the surface and its plane vertical. Find the ratio between the pressures on the two areas into which the triangle would be divided by a horizontal line through its center of pressure.

13. Compute the stress in a 36-inch pipe, whose walls are $\frac{3}{8}$ in. thick, if the water which fills it is under a pressure equivalent to 230 ft. of head on its center.

Ans. 4800 lb. per square inch.

14. A wood stave pipe, 48 in. in inside diameter, is to resist a maximum water pressure of 150 lb. per square inch. If the staves are bound together by flat steel bands, 4 in. wide by $\frac{1}{4}$ in. thick, find the spacing distance of the latter in order that they may not be stressed above 15,000 lb. per square inch.

Ans. 12.5 in.

15. If a 12-inch flanged pipe be closed at its end by a hemispherical cap bolted to the flange, what total stress will be in the bolts when the head on the pipe's center is 240 ft.?

Ans. 11,800 lb.

16. What thickness should be given the steel wall of a 60-inch pipe if it is to withstand a pressure of 100 lb. per square inch with a maximum fiber stress of 15,000 lb. per square inch?

Ans. 0.2 in.

17. A timber dam has a plane upstream face sloping at an angle of 60 degrees with the horizontal. Compute the vertical and horizontal components of the pressure (per linear foot) against it when water stands 30 ft. deep behind the dam. Compare these figures with those obtained for a slope of 45 degrees.

Ans. (*a*) V = 16,250,
H = 28,150.
(*b*) V = 28,150,
H = 28,150.

18. The flat bottom of a steel tank is connected with the plane, vertical side by a plate curved through 90 degrees on a radius of 2 ft. If water stands to a depth of 8 ft. in the tank, compute the horizontal and vertical components of the normal pressure on a linear foot of the curved plate.

Ans. V = 939 lb., H = 876 lb.

19. A tank with vertical sides is 4 ft. square, 10 ft. deep, and is filled to a depth of 9 ft. with water. By how much, if at all, will the pressure on one side of the tank be changed if a cube of wood, specific gravity 0.5, measuring 2 ft. on an edge, be placed in the water so as to float with one face horizontal?

Ans. 780 lb.

20. A cask which weighed 60 lb. was placed on platform scales and then nearly filled with water. A total load on the scales of 320.25 lb. was read.

Should the net weight of water as computed from these figures be corrected by reason of the fact that a 3-inch vertical steel shaft suspended from the ceiling above had its lower end immersed in the water to a depth of 1 ft.? If so, by what amount? *Ans.* 3.06 lb.

21. A rectangular caisson is to be sunk, in which to build the foundation for a bridge pier. It is in the form of an open box, 50 ft. by 20 ft. in plan, and 23 ft. deep. If it weighs 75 tons, how deep will it sink when launched? The water being 20 ft. deep, what additional load will sink it to the bottom? *Ans.* 2.4 ft.
550 tons.

22. A flat-bottomed scow is built with vertical sides and straight sloping ends. Its length on deck is 80 ft., on the bottom 65 ft., its width 20 ft., and its vertical depth is 12 ft. How much water will it draw if it weighs 250 tons? *Ans.* 5.8 ft.

23. A ship with cargo weighs 5000 tons and draws 25 ft. of water. On crossing a bar at the entrance to a river her draft is decreased by 1 ft. by the discharge of 300 tons of water ballast. In going up the river to fresh water, 50 tons of coal are burned. What will her draft be then and how much ballast will be required to increase it by 1 ft.? Assume the unit weight of salt water to be 64.0 lb. (Suggestion: The shape of the vessel's under body is not known and the problem should be solved on the basis of displacement in cubic feet. The sides of the vessel near the water line may be assumed vertical.)

Ans. 24.21 ft.
293 tons.

24. A ship with a total displacement of 1800 short tons rolls to one side through an angle of 1 degree when a deck load of 5 tons is moved laterally through a distance of 15 ft. Compute the metacentric height for this particular position of the vessel.

Ans. 2.39 ft.

25. A rectangular pontoon is 80 ft. long, 40 ft. wide, and 12 ft. deep. Its draft when launched is 4 ft. and is increased to 10 ft. when fully loaded. Compute the position of the true metacenter for both drafts. Assume in each case the center of gravity of the pontoon and load at the geometrical center of the cross-section.

Ans. 29.3 ft., light.
12.4 ft., loaded.

26. A hollow cylinder 3 ft. long and 3 ft. in diameter, closed at one end, is immersed with axis vertical and closed end uppermost, this end being held 25 ft. below the water surface. The cylinder is at first full of water, but compressed air is admitted from beneath in the immersed position until it has displaced two thirds of the water.

Find:

(*a*) Absolute pressure from above on top of cylinder.
(*b*) Absolute pressure from below on top of cylinder.
(*c*) Supporting capacity of cylinder due to buoyancy.
(*d*) Supporting capacity of cylinder if it be allowed to rise 15 ft.
(*e*) Position of maximum supporting capacity with same charge of air.

Ans. (*a*) 26,040 lb. (*b*) 26,940 lb. (*c*) 900 lb.
(*d*) 1155 lb. (*e*) 3.6 ft. head on top.

27. If the specific gravity of a body is 0.8, what proportional part of its total volume will be above the surface of the water upon which it floats?

Ans. 0.20.

28. A stick of yellow pine timber, weighing 40 lb. per cubic foot, measures $6'' \times 12'' \times 20'$. What load will it carry without sinking, if placed in fresh water?

Ans. 225 lb.

29. Two vessels, A and B, containing water under pressure, are connected by an oil differential gauge of the type illustrated in Fig. 9. If a point m in A is 4.85 feet below a point n in B, find the difference in pressure at these points when the top of the water column in the tube entering A stands 15 inches below that in the tube entering B. Specific gravity of oil is 0.80.

Ans. 2.0 lb. per square inch.

30. A differential gauge is used to measure the air pressure in a ship's stokehold. It is in the form of a vertical, glass U-tube, partly filled with water, having one leg in the stokehold and the other outside where the pressure is atmospheric. Oil with a specific gravity of 0.80 fills the inside leg, the line of separation between oil and water being in this leg. The upper ends of the tube are enlarged so that the sectional area is 10 times that of the rest of the tube, and the top surfaces of the oil and water are in the enlarged portions. Find what increase in pressure (over that of the atmosphere outside) in the stokehold will force the line of separation downward 1 inch.

Ans. $p = 0.014$ lb. per square inch.

CHAPTER III

1. What distance must the sides of a tank be carried above the surface of water contained in it, if the tank (moving horizontally) is to suffer an acceleration of 10 ft. per second each second without losing water? Tank is 6 ft. square with water 3 ft. deep. Compute the maximum intensity of pressure on the bottom of the tank during acceleration.

Ans. (*a*) 0.93 ft.
(*b*) $p = 193$ lb. per square inch.

2. A vessel containing oil (specific gravity 0.70) moves in a vertical path with an acceleration of 8 ft. per second each second. Find the intensity of pressure at a point in the oil 3 ft. beneath its surface when,

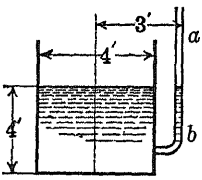
(*a*) moving upward with positive acceleration.
(*b*) moving upward with negative acceleration.
(*c*) moving downward with positive acceleration.
(*d*) moving downward with negative acceleration.

Ans. (*a*) 164 lb. per square foot. (*b*) 123 lb. per square foot.
(*c*) 123 lb. per square foot. (*d*) 164 lb. per square foot.

3. (*a*) According to the principles discussed in Chapter III, what pressure should be found to exist between the water particles forming a jet from a nozzle discharging into the air?

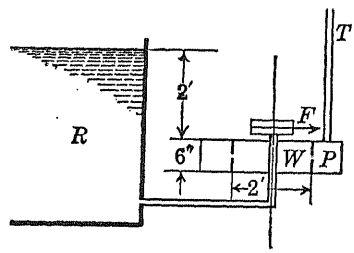
(*b*) Ascertain, by aid of principles discussed in Chapter III, the law of variation in pressure between the top and bottom of a stream of water flowing under pressure through a horizontal pipe. Does the same law hold for an inclined pipe?

4. If the water which just fills a hemispherical bowl of 3 ft. radius be made to rotate uniformly about the vertical axis of the bowl at the rate of 30 revolutions per minute, how much will overflow? *Ans.* 19.5 cu. ft.

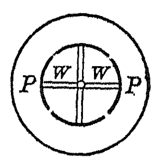


5. The open cylindrical vessel shown in the accompanying sketch is revolved about its center axis at the rate of 56 R. P. M. If previously filled with water to the brim, how high above the latter will water rise in the attached piezometer tube, *a–b*? *Ans.* 2.68 ft.

6. At what speed must an open, vertical cylindrical vessel, 4 ft. in diameter, 6 ft. high, and filled with water, be rotated on its own axis in order that the effect of rotation will be to discharge sufficient quantity of water to uncover a circular area on the bottom of the vessel 2 ft. in diameter. *Ans.* 108 R. P. M.



7. Water from the constant-level reservoir, *R*, flows to, and entirely fills, the closed, concentric cylindrical chambers shown in the figure. A wheel, *W*, composed of flat vanes and driven by a motor, causes the water in the central chamber to rotate as a mass at the rate of 240 R. P. M. With free communication between the central and outer chamber, *P*, how high will water stand in the open piezometer tube, *T*? *Ans.* 100.0 ft.



nean pressure of 60 lb. per square inch. A loss of 0.20 ft. head is exrienced in going through the reducer. What axial thrust on the pipe .e does the water develop as it passes the reducer?

Ans. 33,800 lb.

9. The buckets of an impulse wheel receive water from a 2-inch jet iich has a velocity of 60 ft. per second. If the linear speed of the ckets be 40 ft. per second, and the bucket angle, α, be 170°, find the turnz force applied to the wheel, assuming all water coming from the jet to be ilized. At what speed should the buckets be made to move in order that e power output of the wheel should be a maximum? What would the wer be?

Ans. (*a*) 101 lb.; (*b*) 30 ft. per second; (*c*) 4540 ft. lb.

10. A ship moves forward under the driving force obtained by steadily mping a stream of water, 1 ft. in diameter, having a velocity relative to e ship of 100 ft. per second, directly astern. Compute the driving force.

Ans. 15,250 lb.

11. If the velocity of flow in a 2-foot cast-iron pipe ($E = 12{,}000{,}000$) changed in 0.25 sec. from 2 ft. per second to 0 by closing a valve 00 ft. from a reservoir, what probable increase in pressure due to water mmer will be obtained close to the valve? The pipe wall is $\frac{3}{4}$ in. ick.

Ans. 109 lb. per square inch.

12. A 6-ft. pipe conducting water to a number of turbines is made from el plate 0.25 in. thick. When the velocity of flow is 8 ft. per second, quick closing gate operates to stop the flow. What excess pressure is veloped near the gate due to water hammer? What maximum time may taken in closing the gate without diminishing this pressure if pipe be ,200 ft. long. Estimate the probable time that should be taken if the e in pressure is not to exceed 100 lb. per square inch.

Ans. (*a*) 258 lb. per square inch.
(*b*) 7.25 sec.
(*c*) 19 sec.

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